MATH 3TP3 Assignment #9
Due: Friday, November 23, in class

You may make free use of abbreviations defined in lectures, including

$$\begin{array}{cc} ListElement(x,y,z) & Exp(x,y,z) \\ HasLength(x,y) & Concat(x,y,z) \\ Arithmoquine(x,y) & Theorem_S(x). \end{array}$$

You should assume for these questions that we have fixed a Gödel numbering of the language of arithmetic; so e.g. if you want to refer to the Gödel number of the symbol $\vee$, you may do so using the notation $\ulcorner\vee\urcorner$.

1. Find a formula $GodelNumeral(x,y)$ which is true in $\mathbb{N}$ precisely when y is the Gödel number of the term $\overline{x}$.

   (Hint: Look at our definition of Exp.)

2. Using the arithmoquine technique, find a TNT-sentence σ which is true in $\mathbb{N}$ iff

   $$\text{TNT} \vdash \sim\sigma.$$

   Is σ true in $\mathbb{N}$? Can you use σ to show that TNT is incomplete for $\mathbb{N}$?