MATH 3U03 Winter 2015 Midterm 1

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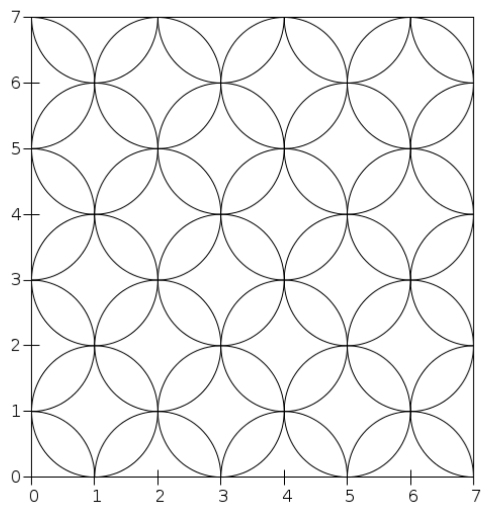
Midterm 1
Duration of test: 50 minutes
McMaster University
2015

Write complete answers to **3** of the **4** questions. Partial credit may be given.
You must justify your solutions to get full marks. You may use results shown in lectures without proving them, but you should make it clear what you are using.
Please be sure to include your name and student number on all sheets of paper that you hand in.
The test is marked out of **15**, each question being worth **5** marks. You are expected to answer three of the questions. You may answer all four questions if you wish, but only your three best marks will count towards the total.

1. [5] How many 7-digit numbers (the integers between 1000000 and 9999999) have no three consecutive digits equal?

2. [5] Consider the square $[0,7]\times[0,7]$. and (partial) discs of radius one with centres (i,j) in the square where i and j are integers with $i+j$ even.



Suppose 73 points are chosen within this square. By considering the above diagram, or otherwise, show that some disc of radius one contains at least three of the points.

3. [5] Prove that for any $n \geq 0$,

$$\sum_{r=0}^{n} \binom{n}{r}\binom{n+1}{r} = \binom{2n+1}{n}.$$

4. [5] Say a pair of integers (a,b) is prime-separated if their difference is a prime number (i.e. $|a-b|$ is prime).

   Prove that for sufficiently large n, if a sequence of n integers has the property that for **any** 5 of the integers, **some** pair amongst the 5 is prime-separated, then for **some** 5 of the integers, **every** pair amongst the 5 is prime-separated.