

1 In binary

011
101
111
001

e.g. take 1 from the first pile,
leaving 2, 5, 7
which has sum 0
so is a second player win.

2. (i) $P(4, 4) = 4!$

(ii) permutations of the multiset

$\{1 \times t, 2 \times e, 3 \times s, 1 \times a, 1 \times y\}$

$$= \frac{8!}{2!3!}$$

3. By the packed pigeonhole principle
with the pigeons as the pigeonholes
and the pigeonholes as the pigeons,
since $50/20 > 3-1$,
some ~~at least~~ pigeon must get ≥ 3 holes.

4. Only ways to get 58 are

(i) 9 6's and one 4 - 10 ways

(ii) 8 6's and two 5's - $\binom{10}{2} = 45$ ways

~~there are~~

so 55 of the 6^{10} possible outcomes yield 58

so prob = $\frac{55}{6^{10}} = 9.1 \times 10^{-7}$