Erratum to "An extremal theorem in the hypercube"

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In the concluding remarks of the paper, we apply the Lovász local lemma to conclude that $ex(Q_n, C_{2t}) = \Omega(2^n n^{\frac{1}{2}+\frac{1}{2t}})$. The application is as follows: if we choose edges of Q_n independently with probability p, the event that any given C_{2t} appears has probability p^{2t} and, for any given C_{2t}, there are $O(n^{t-1})$ other copies of C_{2t} which share an edge with it. Therefore, applying the local lemma, provided $p^{2t}n^{t-1}$ is smaller than some particular constant, we find that a random subgraph of Q_n contains no copy of C_{2t} with positive probability. However, this positive probability is too small to guarantee that the random graph has density close to p and, therefore, the application is invalid.

Instead, one applies the deletion method. The expected number of edges in a random subgraph of Q_n is $p2^{n-1}n$ and the expected number of copies of C_{2t} is $O(p^{2t}2^n n^t)$. Therefore, provided $pn \geq cp^{2t}n^t$ for a sufficiently large constant c, we may delete all copies of C_{2t} from the random graph and still be left with a positive fraction of the edges. This gives a lower bound of the form $ex(Q_n, C_{2t}) = \Omega(2^n n^{\frac{1}{2}+\frac{1}{4t-2}})$.

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