Axioms for Fields

A field is a set F with distinguished elements 0, 1 and with two binary operations $+$ and $\times$ satisfying the following conditions—*the Axioms of Arithmetic*. Conventionally

for the function $+ : F \times F \longrightarrow F$ we write $(a,b) \mapsto a+b$;
for $\times : F \times F \longrightarrow F$ we write $(a,b) \mapsto ab$.

(1) $a + (b + c) = (a + b) + c$ [+ is associative]

(2) $a + b = b + a$ [+ is commutative]

(3) $a + 0 = a$

(4) $(\forall a \in F)(\exists b \in F)(a + b = 0)$

(5) $a(bc) = (ab)c$ [× is associative]

(6) $ab = ba$ [× is commutative]

(7) $a1 = a$

(8) $(\forall a \in F \setminus \{0\})(\exists b \in F)(ab = 1)$

(9) $a(b + c) = ab + ac$ [× distributes over +]

(10) $0 \neq 1$.