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# Preferred aspect ratios of convection in a strongly temperature and pressure dependent viscous fluid

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If it is assumed that mantle convection is shallow, i.e. limited to an upper layer of ~ 700 km, then the extent of surface plates must be such that aspect ratios are noticeably large, in contrast to many laboratory and numerical experiments. A rheology which depends strongly on temperature and pressure is provided as an explanation of why this may occur. The aspect ratio, it is suggested, is preferentially large because of the unwillingness of the stiff plates to subduct, which raises the problem of how subduction occurs. A hypothesis is proposed that this is caused by viscous heating in the asthenosphere, and the preferred aspect ratio is large enough such that partial melting takes place *underneath* the sinking slab, causing it to sink by releasing the sub-lithospheric pressure so that a transverse buckling may occur.

## 1. Depth structure of mantle convection

In a companion paper to this (Fowler, 1982a), I have argued that for a material whose viscosity depends strongly on temperature and pressure, the preferred depth of convection cells is small and decreases to zero as the activation energy increases to infinity. Furthermore, application of this purely mechanical reasoning to the parameters relevant to mantle convection shows that a convective depth of $\leq 700$ km is certainly consistent with the theory. Shallow mantle convection receives support in the literature from the limited depth of deep earthquakes, the seismic discontinuity at ~ 650 km that possibly reflects a chemical discontinuity as well as a phase change (Anderson, 1979), and the layering of the mantle which is apparently necessary to explain the different chemistry of ocean island basalts and mid-ocean ridge basalts. Mechanical arguments for whole mantle convection (Elsasser et al., 1979) have been based on the notion of a constant viscosity mantle (Cathles, 1975) and the apparently better agreement between quantitative results of boundary-layer theory (Turcotte and Oxburgh, 1967; Olson and Corcos, 1980) and a mantle convecting as a single layer, when measured heat flow data (Parsons and Sclater, 1977) is taken into account. On the other hand, O'Connell and Hager (1980) suggest that this argument is misplaced, since the relevant scale is the length of a convection cell and not the depth.

With shallow convection cells, aspect ratios (of large scale circulation) of the order of 10 exist in the Earth. One cannot easily rule out more complicated possibilities, such as the existence of a number of sub-lithospheric cells of aspect ratio ~ O(1), although the existence of such cells (some rotating in the opposite sense to the plate motion) would require a thermal buoyancy to drive them, either from basal boundary-layer instability (Yuen and Peltier, 1980) or along the lines suggested by Parsons and McKenzie (1978), comparable to the

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negative buoyancy of the descending slab. Apart from thinking this at first sight unlikely, the a priori philosophical attitude is to analyse the simplest models first.

Thus, if aspect ratios are supposed to be large in the Earth, it is natural to expect that even steady two-dimensional models exhibit the same preferential geometry, if they are to be realistic. The question thus arises as to why such large aspect ratios should be naturally preferred. This problem has been addressed by others, either numerically (De Bremaecker, 1977; Hewitt et al., 1981; Schmeling and Jacoby, 1981), or analytically (Richter and Daly, 1978; Chapman et al., 1981). There is no real problem in finding large aspect ratios, but the numerical models fail to simulate the mantle in other ways, and the analytic models are essentially linear, thus fundamentally incomplete. It is felt that a more thorough understanding of aspect ratios requires a description of thermal boundary-layers in a convecting, thermoviscous fluid, and this is, by its nature, an extremely nonlinear problem.

Numerical (e.g., Daly, 1980), theoretical (Fowler, 1982b, Morris, 1981), and experimental (Nataf and Richter, 1981) work suggest that the effect of a temperature dependent rheology is to thicken the top, cold thermal boundary-layer, to make thinner the basal hot layer, and also, to decrease the basal temperature jump in comparison with the upper one, thus taking up the brunt of the boundary-layer structure. Such behavior may be simply understood in terms of an "unstable boundary-layer" (Howard, 1966); if the sinking of the cold boundary-layer is equated with the attainment (locally) of a critical Rayleigh number based on boundary-layer thickness (and similarly for the rising of the basal, hot boundary-layer), then consideration of the marginal stability of the conductive temperature profile of a thermoviscous fluid (Schubert et al., 1969) suggests that only a basal temperature jump $\lesssim RT_b^2/E^*$ can be maintained; where R is the gas constant, $T_b$ is basal (prescribed) temperature and $E^*$ is activation energy. In the Earth, reasonable estimates are $T_b \sim 2000$ K, $RT_b/E^* \sim 1/30$, so that basal temperature jumps much greater than $\sim 60$ K are unlikely to occur. Anything greater than this leads to a boundary-layer which is effectively inviscid in comparison to the material above it, and a catastrophic Rayleigh–Taylor type of instability would preclude the existence of a stable, steady boundary-layer of this type. The fact that the overlying mantle is (very) viscous does not prevent such instability; it simply dictates the growth rate of basal "blobs". Note that this concept is related to that of the highly unstable boundary-layer (Yuen and Peltier, 1980), but here one is led, *by the statement that a major basal boundary-layer would be highly unstable*, to suppose that such a layer will simply not exist. This will certainly be true in a steady state and corresponds to experimental results of Nataf and Richter (1981).

In an exactly similar manner, the top cold thermal boundary-layer is stiffer than the material beneath and hence is less willing to sink. Additionally, the lighter composition (basalt, harzburgite) of the lithosphere in comparison to the underlying mantle (garnet peridotite) will act to keep the former buoyantly stable. In fact, since the viscosity is strongly exponentially dependent on temperature, it seems unlikely that the boundary-layer will ever become unstable, since as the boundary-layer thickens, the effective viscosity of the layer increases exponentially, so that the effective Rayleigh number decreases with age: in other words, increasing (destabilising) heaviness of the layer is overcompensated by rapidly increasing (stabilising) stiffness due to the exponential rheology. In itself, this appears to provide a qualitative reason why large aspect ratios are preferred: the surface boundary-layer is simply unwilling to subduct, because of its effective rigidity. Apparently, the boundary-layer would eventually thicken to fill the whole convecting layer (which would be inconsistent), but in fact this is only the case if it is moving, which presumes a subduction zone a priori. There is a perfectly feasible alternative structure, and that is a stagnant conductive plate at the surface, with convection occurring underneath. The style and aspect ratio of such convection will probably be entirely different to that with active plates, but in any case is beyond the present concern. Such a structure is evident in the studies of Pearson (1977), Ockendon and Ockendon (1977) (see also Ockendon, 1979; Morris, 1981) and is glimpsed in some of the numerical results of Daly (1980) and Schmeling and Jacoby (1981). Since the stiffness of the top boundary-layer seems to want a mode of convection with stagnant plates, such as presumably occurs in the Moon and other terrestrial planets, the question now arises, what makes the lithosphere subduct at all?

## 2. The mechanism of subduction

Before recalling two proposed mechanisms of subduction, the likely functions of the model should be considered. Physically, purely fluid equations with a thermoviscous rheology would apparently have a stable solution with stagnant plates. Non-fluid effects, such as elastic or plastic behavior, would have a regularly perturbing effect on this solution. Therefore, in order for subduction to occur, it is necessary that this purely fluid solution should be unstable, due to purely fluid dynamical processes. This does not mean that subduction is hydrodynamic, but it is envisaged that the fluid instability of a stagnant plate solution leads to a situation where non-fluid effects (partial melting, elasticity, fracturing, etc.) cannot consistently be ignored and that subduction then occurs by such processes. However, the important point is that the initiation should be controlled by fluid dynamics in the rest of the cell.

Some constraints have to be considered on a steady-state convecting cell in which subduction occurs. Attention is drawn to two of the various definitions (Anderson and Minster, 1980) of the "lithosphere". Firstly, the elastic lithosphere, with a depth of 30 km, is that part of the upper mantle which deforms essentially by elastic, rather than viscous, deformation. Secondly, the mechanical lithosphere is the surface layer which behaves effectively rigidly by comparison with the underlying convecting flow. In view of the previous comments and also those of Fowler (1982a), this may be considered effectively identical to the thermal lithosphere (which is just the thermal boundary-layer), since the viscosity reaches its adiabatically determined (approximately) isoviscous state just at the edge of the layer. Above this, variations in temperature of $O(T_b)$ lead to exponentially large viscosity (because $E^*/RT_b \gg 1$), i.e. a mechanically "rigid" lid.

In a steady state convection cell (which, it is emphasised, is considered to be of relatively "shallow" depth), the coolth accumulated in the top thermal boundary-layer must either be removed in the downwelling (subduction) zone, or be convected into the basal layer. The former is the case in Turcotte and Oxburgh's (1967) analysis. If the aspect ratio is large, then conservation of mass implies (1) either a broad downwelling zone of cold material (which would then presumably behave quasi-rigidly, in contrast to the observed narrowness of seismic subduction zones), or (2) convection of this material into the basal layer (as observed in numerical calculations of a newtonian convection by Hewitt et al. (1981), or (3) the descending thermal boundary-layer must be thinner than the surface one. No other alternative seems feasible. Of these three, the first is ruled out, and so is the second, since there is no obvious way a "rigid" slab can be bent round at depth. At the surface, gravity helps to make the slab founder (see below): at the base, there is no obvious reason why a thick, cold, descending slab should decide to turn through 90 degrees rather than simply stop and become stagnant. (Implicitly, this reasoning is part of the rationale behind the objections of Elsasser et al. (1979) to shallow mantle convection.) Until an ingenious non-hydrodynamic explanation for such slab reversal is tendered, then the third possibility must be considered.

If the thermal boundary-layer in the downwelling slab is warmed by the time it reaches a depth of ~ 700 km, then, since conservation of mass implies that the velocity of the slab is equal to that of the plate (at least in a steady state, and/or when the trench is stationary with respect to the ridge: even if the latter case is invalid, the velocities will be asymptotically comparable, and the argument proceeds), scaling laws immediately imply that this boundary-layer must be of thickness $\nu^{1/2}$ times the thermal lithospheric thickness (Fowler 1982b), where $1/\nu$ is the aspect ratio. If $1/\nu \sim 10$ and the thermal lithosphere is ~ 100 km thick, then the slab width is ~ 30 km, coinciding with the elastic plate thickness, and the observed width of Benioff zones under Japan (Uyeda 1977).

Apparently, the proposition left is that, if large aspect ratio cells occur in shallow mantle convection, then (at least in any conceptual steady state) the temperature deficit in the ~ 100 km thick lithosphere as it arrives at a trench is converted into a deficit across only ~ 30 km: warming must occur near a trench. This seems to be a constraint which a steady state subducting model of this type should satisfy. However, Wortel (1982) has pointed out in a recent paper (see also Molnar et al., 1979) that a simple and reasonable explanation for both the cessation of deep earthquakes at ~ 700 km and the observed width of Benioff zones may be understood in terms of a critical temperature $T_{cr}$ (depending on depth) which determines (roughly) the switch-over from viscous to elastic (and hence potentially seismic) behavior; he obtains favourable agreement with observations by using a slab cooling model which reaches temperatures ~ 1250 K at a depth ~ 700 km. This is much colder than one would expect from upper mantle convection, and can be used as a serious argument in favour of whole mantle convection. Wortel's arguments combine a series of simple models (slab cooling, Maxwell criterion for the critical temperature (Caldwell and Turcotte, 1979), and assumed geotherm) and may be criticised on this basis. However, his results represent a problem which must be seriously addressed by those in favour of upper mantle convection. Nevertheless, since the theme of this paper is to examine the feasibility of an assumed shallow convection cell from the point of view of thermoviscous fluid mechanics (a somewhat different matter), this line of argument will not be pursued here.

McKenzie (1977) argued that a collision between two plates could lead to a finite amplitude buckling instability, followed by underthrusting and subsequent foundering. Compressive stresses $\gtrsim 800$ bars and relative velocities $\gtrsim 1.3$ cm $y^{-1}$ are required for this mechanism. The difficulty with this explanation is that, whereas it is a viable mechanism once subduction is already occurring elsewhere, there does not seem any way it can induce initiation of subduction on a Earth with an unbroken stagnant lithosphere. Stresses in the order of kilobars can occur by membrane tectonics (Turcotte, 1974) if the outer skin rotates relative to the rotation axis, for example as a result of true polar wander, but relative velocities appear to require the prior existence of plates. Additionally, phenomena such as high heat flow behind trenches and island arc formation are not specifically catered for.

Turcotte et al. (1977) argued that subduction could occur by the transverse buckling of the elastic lithosphere. To initiate such foundering, they propose that uprising diapirs of hot mantle rock could occur because of the manometer-like increase of pressure, with depth, through the lithosphere. Essentially, subsidence of the ocean floor with age leads to an increase of sub-lithospheric pressure which may become sufficient to drive partially molten diapirs to the surface. Eventually, such loading on the ocean floor leads to elastic failure and hence subduction. This explanation is more attractive than the other, but nevertheless does not answer why hot, partially molten rock should occur away from a ridge beneath old and cool lithosphere.

## 3. Hydrodynamic effects

It is recalled that, ideally, the initiation of subduction should occur as a result of a purely hydrodynamic instability, possibly leading to the process outlined by Turcotte et al (1977); presumably, subduction will continue to occur by this same mechanism. In the discussion below, primary consideration is given to this latter phenomenon, i.e. the continuing process of subduction, rather than to the specific initiation process, which in principle should then be treated in a corresponding analysis of stagnant plate convection. A comment on initiation is given at the end of the paper.

In recent years, many authors (Anderson and Perkins, 1974; Melosh, 1976; Yuen and Schubert, 1977, 1979; Schubert and Yuen, 1978; Melosh and Ebel, 1979) have raised the question of whether the exponential dependence of viscosity, and hence also shear heating, on temperature could cause the existence of multiple steady solutions for the boundary-layer temperature equation, by analogy with similar situations which occur in polymer flow, chemical reactors and other applications (for a review of work to date, see Yuen and Peltier, 1980). Conclusions of these studies are somewhat tentative, and this may be due to the idealized nature of some of the models studied, and the difficulty of interpreting the results physically. A closer consideration of the *concept* of boundary layer instabilities is therefore useful.

Developing boundary-layers in a convecting mantle with active plates may be looked at from two viewpoints, the Lagrangian and the Eulerian. In the Eulerian framework, the time evolution of (for example) the temperature at a fixed point in space is studied, whereas in a Lagrangian framework a material element is followed and the time development of its individual temperature is traced. Either viewpoint may be useful, depending on the context.

The Lagrangian frame is a natural one to study problems in which there are no natural geometric length scales, for example the development of instability in flow past a flat plate, which may be taken as semi-infinite, or the instability of developing boundary-layers in the inlet region of Poiseuille flow, where, in some sense, the instability develops with movement of the fluid. In the case of convection, however, there is a natural geometric length scale: the length of the presumed convective cell. Therefore, *even though boundary-layers naturally occur* at high Rayleigh numbers, the proper framework to view instability of the entire cell is the Eulerian one, and it is a boundary-value problem. That such *global* instabilities manifest themselves as boundary-layer variations (Moore and Weiss, 1973) is simply indicative of the fact that this is where all the interesting temperature structure is. Thermal boundary-layer stability *can* be viewed in a Lagrangian sense, and this is really Howard's (1966) view (though in a slightly different context). Here, however, the developing boundary-layer becomes (in a Newtonian fluid) buoyantly unstable when it is (marginally) heavy enough to sink through the lighter, but viscous, fluid beneath: this is exactly what happens at downwellings, where a "Lagrangian instability" exists and is indeed an essential ingredient of the steady state flow (as well as the unsteady one).

All of the papers cited above are concerned with the stability of various, essentially one-dimensional, problems. To interpret these physically, I would like to suggest they are really aiming at the kind of Lagrangian instability discussed above; i.e. the stability of the entire cell is not being addressed but rather the stability of the thickening lithosphere as it approaches a "side-wall"—or trench. In this light, the infinitely long boundary-layer would *always* become unstable, because the viscous heating term in the boundary-layer exists in an effectively infinite vertical (boundary-layer) domain (see Ockendon, 1979, p. 739 for a similar argument). Thus it is proposed that the presence of viscous dissipation is sufficient to provide a runaway in temperature within the boundary-layer, provided the aspect ratio is large enough. In the lithosphere the shear stress is determined from the momentum equation by the temperature anomaly and the runaway will then presumably occur in a thin layer where the viscosity is at a minimum with respect to depth, i.e. just beneath the rigid plate. This region may be identified as the asthenosphere, and it is suggested that this corresponds to the Low Velocity Zone (LVZ) of seismic studies. (Partial melting also occurs underneath ridges, though in this case it may be thought of as being due to the intersection of the geotherm with the steeper solidus. This is not really a runaway in the same sense, though it does enable large deformations to occur just beneath the ridge.)

What are the implications of such a runaway? The thermal boundary-layer becomes thinner; extensive partial melting occurs, leading to diapiric uprise, pressure release and subsequent foundering of the remaining elastic plate, as proposed by Turcotte et al. (1977). Once subduction is initiated, it is maintained by both the sinking slab (for the incoming elastic plate) and the viscous heating runaway (to thin the thermal boundary-layer). Extensive partial melting continues under the slab, leading to island arc formation and high heat flow behind the trench (a low zone of heat flow very near the trench may be explained by the presence of hydrous minerals absorbing some frictional heat as the plate descends (Anderson et al., 1976)). Therefore, the constraint of thinning of the thermal boundary-layer, and also the production of partial melt, are both essential constituents of this mechanism.

**4. Preferred aspect ratio**

The hypothesis presented above shows how to find the natural aspect ratio for "active" thermo-viscous convection, i.e. the aspect ratio $1/\nu$ is chosen so that when the boundary-layer equations are appropriately scaled, the viscous dissipation term is such that runaway occurs just at the side-wall (trench). Scaling is not a trivial task and requires an analysis of the entire cell which has been done elsewhere (Fowler 1982b). By analogy with the results of Gruntfest (1963), the aspect ratio is chosen to be such that viscous dissipation balances conduction and advection in the thermal boundary-layer equation. Balancing conduction merely dictates the thermal boundary-layer thickness, so the relevant terms in the energy equation are

$$\rho c_p u T_x + \ldots = \ldots + \tau^2/\eta \tag{1}$$

where $\rho$ is density, $c_p$ is specific heat, $u$ is horizontal velocity, $T$ is temperature, $x$ is horizontal distance, $\tau$ is stress ($\tau^2$ is the second stress invariant) and $\eta$ is the viscosity, given by an expression of the form (Goetze, 1978)

$$\eta = \left[A/(\tau^{n-1})\right] \exp\left[(E^* + pV^*)/RT\right]. \tag{2}$$

To write (1) in dimensionless form, $T_a$ is taken as a typical asthenospheric temperature; put

$$T = T_a\theta,\ x = l\xi, \tag{3}$$

where $l$ is the length of the cell under consideration. The viscosity can then be written as

$$\eta = \eta_a \exp\left[(1-\theta)/\epsilon\theta\right], \tag{4}$$

where $\eta_a$ is a typical asthenospheric viscosity (dependence on $\tau$ and $p$ is suppressed),

$$\varepsilon = RT_a/E^*, \tag{5}$$

and $\theta \to 1$ just outside the boundary-layer can be ensured by appropriate choice of $T_a$. The boundary-layer version of (1) is then

$$\partial\theta/\partial\xi + \ldots = \ldots + \lambda \exp\left[(\theta - 1)/\varepsilon\theta\right], \tag{6}$$

where

$$\lambda = l\tau^2/\rho c_p T_a u \eta_a \tag{7}$$

is a dimensionless measure of the importance of viscous heating. To balance (6), it is firstly noted that a typical estimate of $\varepsilon$ is $\varepsilon \sim 1/30 \ll 1$, and secondly that the exponential term is only of relevance while $\theta - 1 \sim O(\varepsilon)$ (see also Pearson, 1977). Thus define

$$\theta = 1 + \varepsilon\phi, \tag{8}$$

obtaining from (6), at leading order in $\varepsilon$,

$$\partial\phi/\partial\xi + \ldots = \ldots + \Lambda \exp(\phi), \tag{9}$$

with

$$\Lambda = \lambda/\varepsilon = l\tau^2 E^*/\rho c_p \eta_a u R T_a^2. \tag{10}$$

Bearing in mind that (9) will also contain a diffusion term, this very much resembles Gruntfest's (1963) problem, and it can be supposed that the length scale $l$ on which runaway takes place is determined by choosing

$$\Lambda \sim 1,\ \text{i.e.}\ l \sim \rho c_p \eta_a u R T_a^2/E^*\tau^2. \tag{11}$$

If $\rho = 3.5$ g cm$^{-3}$, $c_p = 0.27$ cal g$^{-1}$ K$^{-1}$, $\eta_a = 4 \times 10^{20}$ Poise, $u = 10$ cm y$^{-1}$, R $= 8.3$ J mole$^{-1}$ K$^{-1}$, $T_a = 1500$ K, $E^* = 122$ kcal mole$^{-1}$, $\tau = 10$ bars, then

$$l \sim 19000\ \text{km}. \tag{12}$$

Obviously, the parameters can be altered to tune $l$ up or down, but (12) is of the right order of magnitude and as far as a rough order of magnitude goes, is completely satisfactory. To calculate smaller values of $l$, the following can be taken: $\tau > 10$ bars, $u < 10$ cm y$^{-1}$, $\eta_a < 4 \times 10^{20}$ Poise, and $\Lambda > 1$, since diffusion can be expected to increase the runaway distance $\xi = 1/\Lambda$ of the diffusionless equation $\phi_\xi = \Lambda e^\phi$, $\phi|_{\xi=0} = 0$.

An additional point to note from (10) is that runaway can occur on a much smaller scale if $\tau$ is larger. Since constant viscosity analysis at high Rayleigh numbers (Roberts 1979) reveals stress concentrations at stagnation points due to the abrupt transition from a low stress horizontal thermal boundary-layer to a high stress (e.g., ~ 500 bars) quasi-vertical vorticity layer (downwelling), the same behaviour can be legitimately expected in the mantle; putting $\tau = 1000$ bars then determines $\Lambda \sim 1$ when $l \sim 20$ km, with the same values as before. This may have a bearing on the continuing process of subduction and is clearly of relevance to the problem of initiation: the effect is accentuated by the dependence of $\eta$ on $\tau$.

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