Temperature Surges in Thermistors

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Abstract:

A thermistor is a nonlinear resistor whose resistivity increases with temperature. We analyse a simple circuit containing such a device and show that under certain circumstances rapid temperature surges can occur.

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1. Introduction

Thermistors are circuit components made from a ceramic material whose electrical resistivity $\rho(T)$ varies significantly with temperature T. In this paper we discuss the interaction between the heat generated in the device and the current flow through it, and the subsequent change in the current itself. In particular, we consider the behaviour of a positive-temperature-coefficient thermistor, i.e. one whose resistivity increases with temperature (as opposed to a negative-temperature-coefficient device whose resistivity decreases with increasing temperature). Such devices are frequently used to protect circuits, since any current surge leads to a temperature increase which in turn reduces the current by increasing the thermistor's resistance.

A simple circuit is shown in Fig. 1a. A short circuit is represented by closing the switch S, and we require a model for the subsequent evolution of the current. Two questions are of particular interest: (a) what is the dependence of the steady current I_0 on the external voltage V_0, and (b) can large temperature gradients occur inside the device. The second question is prompted by the experimental observation [1] that for large values of V_0 the device can crack; it is suspected that this may be caused by thermal stresses associated with high temperature gradients.

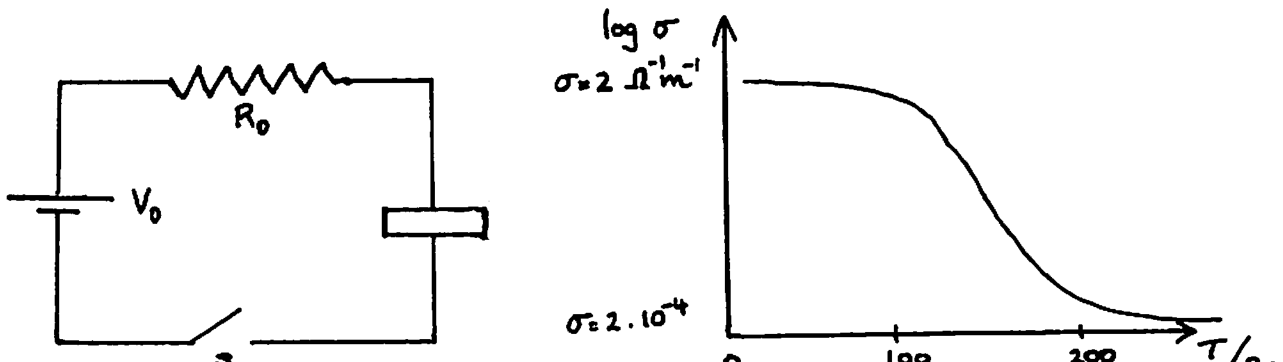

Fig. 1

(a) (b)

It is convenient to work with the electrical conductivity $\sigma(T) = 1/\rho(T)$; the variation of $\log\sigma$ with T is sketched in Fig. 1b. The main result of this paper is to show that the temperature variation of σ and the external resistance R_0 can combine to produce large temperature gradients within the device. This fact was noted in [2]; previous work (see [2] for a review and references) had always ignored the effect of the external circuit.

In form, a typical thermistor is a cylinder whose thickness 2H is about 2mm, and whose radius is about 5mm. The two end surfaces are covered in thin metal contacts and onto these are soldered connecting wires. The net effect of this arrangement is that the heat loss from the thermistor is mostly through the top and bottom, and thus for simplicity we consider a one-dimensional model. With distance x measured from the centre plane, the temperature $T(x,t)$ and electric potential $\Phi(x,t)$ satisfy, for $t > 0$ and $|x| < H$, conservation of charge:

$$\frac{\partial}{\partial x}\left(\sigma(T)\frac{\partial\Phi}{\partial x}\right)=0 , \qquad (1.1)$$

and conservation of heat: $$\bar{\rho} c \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2} + \sigma(T)\left(\frac{\partial\Phi}{\partial x}\right)^2 ; \qquad (1.2)$$

the last term in (1.2) represents the Joule heating.
On the conducting surfaces $x = \pm H$, the potential satisfies

$$\Phi(\pm H,t) = \pm\Phi_0(t) \qquad (1.3)$$

where Φ_0 satisfies the circuit equation

$$V_0 = I(t)R_0 + 2\Phi_0(t) , \qquad (1.4)$$

in which the current $I(t)$ is given by

$$I(t) = \pi r^2\left(\sigma(T)\frac{\partial\Phi}{\partial x}\right)\Big|_{x=H} . \qquad (1.5)$$

At t = 0, the temperature satisfies

$$T(x,0) \equiv T_a \tag{1.6}$$

where $T_a \sim 20^{\circ}C$ is the ambient temperature; on $x = \pm H$, we model the heat transfer to the surroundings by a heat transfer law,

$$\pm k \frac{\partial T}{\partial x} + h(T-T_a) = 0 \tag{1.7}$$

where h is a heat transfer coefficient. Lastly, we assume that the variation of σ with T is given by an exponential law:

$$\sigma = \sigma_0 \exp[-F(T)] \tag{1.8}$$

where $\sigma_0 = \sigma(T_a)$ is the 'cold' value of σ and

$$\begin{aligned} F(t) &= 0 & T_a < T < T_a + \Delta T \\ &= \frac{T - (T_a+\Delta T)}{\varepsilon \Delta T} & T_a + \Delta T < T < T_a + 2\Delta T \\ &= -1/\varepsilon & T_a + 2\Delta T < T < \infty . \end{aligned}$$

where $\Delta T \sim 100^{\circ}C$ is the increase in temperature needed before σ starts to decrease, and $\varepsilon \sim 10^{-1}$ is dimensionless.

Typical values for the parameters in these equations are as follows[1]:

$\bar{\rho}c = 3 \times 10^6 \text{ J m}^{-3} \text{ K}^{-1}$

$k = 2 \text{ W m}^{-1} \text{ K}^{-1}$

$H = 10^{-3} \text{ m}$

$r = 5 \times 10^{-3} \text{ m}$

$h = 10^2 \text{ W m}^{-2} \text{ K}^{-1}$.

$\sigma_0 = 2 \text{ m}^{-1} \ \Omega^{-1}$

$R_0 = 50\,\Omega$

$V_0 = 250\text{V}$

$\Delta T = 100\text{K}$.

Using these parameters, and with some hindsight, we scale the variables as follows:

$$x = L\,\bar{x} \qquad t = (H^2\rho c/k)\,\bar{t}\ , \quad T - T_a = (\Delta T)u$$

$$\Phi = \tfrac{1}{2}V_0\phi \qquad \sigma = \sigma_0\,\bar{\sigma}\ .$$

Dropping the overbars, we have the following dimensionless system:-

For $\quad - 1 < x < 1, \qquad t > 0,$

$$(\sigma(u)\phi_x)_x = 0, \tag{1.9}$$

$$u_t = u_{xx} + \gamma\sigma(u)\phi_x^2\ ; \tag{1.10}$$

on $\quad x = \pm 1, \quad \pm u_x + \beta u = 0;$ (1.11)

and $\quad \phi = \pm (1-\lambda\phi_x)\ ;$ (1.12)

and $\quad u(x,0) = 0\ .$ (1.13)

The variation of $\sigma(u)$ with u is given by

$$\sigma(u) = \exp\,(-f(u)/\varepsilon) \tag{1.14}$$

where

$$f(u) = \begin{cases} 0 & 0 < u < 1 \\ u-1 & 1 < u < 2 \\ 1 & 2 < u < \infty\ . \end{cases} \tag{1.15}$$

The dimensionless parameters here are

$$\gamma = \sigma_0 V^2/4k\Delta T \ \simeq 150\ , \qquad \beta = hH/k \simeq 10^{-1}\ ,$$

$$\varepsilon \simeq 10^{-1} \text{ (taken from data for } \sigma\text{)}, \ \lambda = \tfrac{1}{2}\, r^2 R_0\sigma_0/H \simeq 40\ .$$

We analyse (1.9) - (1.15)in the following section.

2. Asymptotic analysis

All the dimensionless parameters in (1.9)-(1.15) are either large or small. In particular, the fact that ε is small suggests using the techniques of high-activation-energy asymptotics. We give a summary here; more details can be found in [2].

We begin by integrating (1.9) and substituting for ϕ_x into (1.10). The result is the non-local equation

$$u_t = u_{xx} + \frac{(\gamma/\sigma(u))}{[\lambda + \int_0^1 \frac{dx}{\sigma(u(x,t))}]^2}$$

$$= u_{xx} + \frac{\gamma \exp(f(u)/\varepsilon)}{[\lambda + \int_0^1 \exp(f(u)/\varepsilon)dx]^2} . \qquad (2.1)$$

with $\partial u/\partial x + \beta u = 0$ at $x = 1$

$\partial u/\partial x = 0$ at $x = 0$ (by symmetry).

There are three stages in the evolution of u.

1. Almost uniform increase in u until time t^*, when $u(0,t^*) = 1$. While $0 < u < 1$, $f(u) = 0$ and so u satisfies $u_t = u_{xx} + \gamma/(\lambda+1)^2$ with $u_x + \beta u = 0$ at $x = 1$. Because β is small, the solution is $u = \gamma t/(\lambda+1)^2(1+O(\beta))$. Thus the dimensionless t^* is approximately λ^2/γ.

2. Acceleration in u near $x = 0$ As soon as u reaches 1 at $x = 0$, the term $\exp(f(u)/\varepsilon)$ in the numerator of the heating term in (2.1) 'switches on' and produces locally large heating in a thin region near $x = 0$. This 'surge', reminiscent of thermal runaway problems, lasts until the integral in the denominator of (2.1) becomes larger than λ and reduces the heating term. The details of the unsteady development of this transition are complicated [2] and will not be considered here; we merely note that the timescale is thought to be $O(\varepsilon\lambda^2/\gamma)$. This is the phase that may cause cracking via thermal stresses.

3. Equilibrium by conduction to a steady state

If the maximum temperature is u^*, the heating term in (2.1) is only effective when $u^* - u \sim O(\varepsilon)$. This suggests that the thickness of the layer where the heating is effective is $x \sim O(\beta/\varepsilon) \sim O(1)$ for our parameter values, and then a balance of terms in (2.1) gives

$$u^* \simeq \varepsilon \ln(\gamma/\varepsilon) + 1 ,$$

which for our parameters corresponds to a realistic maximum temperature of about 190°C.

Our final point concerns the 'current-voltage characteristic', i.e. the dependence of the steady current on V_0. Clearly until u reaches 1 at $x = 0$ the thermistor resistance is unchanged, and so the steady current $I_\infty = V_0/(R_0+R_T)$, where R_T is the thermistor resistance $\pi r^2/\sigma_0 L$. However, when $u(0,\infty) > 1$, which occurs at voltages V_0 such that $\gamma/(1+\lambda)^2 > 2\beta/(2+\beta)$, the current starts to fall. In the steady state described above, the (dimensional) current is easily shown to be approximately

$$I_\infty \sim \frac{V_0}{R_0} \cdot \frac{\lambda}{\lambda + m\gamma/\varepsilon}$$

where m is an $\sigma(\varepsilon/\beta)$ constant depending on the precise details of the temperature profile. Substituting for λ,γ, one finds that since $\lambda << m\gamma/\varepsilon$,

$$I_\infty \sim \frac{4\pi\varepsilon H k \Delta T}{m} \cdot \frac{1}{V_0} ,$$

so that I_∞ changes from being proportional to V_0 to being inversely proportional as V_0 increases.

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References

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