LIBOR model

Let L_i^n denote the forward LIBOR rate for the time interval $[i\,\delta, (i{+}1)\,\delta)$ at time $n\,\delta \le i\,\delta$. Taking the timestep to be equal to the LIBOR interval δ, the evolution of the forward rates L_i^n for $n=0,\ldots,N_{mat}-1$ is approximated by the discrete equations

$$L_i^{n+1} = L_i^n \,\exp\left((\sigma_{i-n-1} S_i - \tfrac{1}{2}\sigma_{i-n-1}^2)\,\delta + \sigma_{i-n-1} Z^n \sqrt{\delta} \right), \quad i>n.$$

where Z^n is the $N(0,1)$ random variable for the n^{th} timestep, and

$$S_i^n = \sum_{j=n+1}^{i} \frac{\sigma_{j-n-1}\,\delta\,L_j^n}{1+\delta\,L_j^n}, \quad i>n.$$

The model treats the volatility as being a function of time to maturity. Once a rate reaches its maturity it remains fixed, so we set $L_i^{n+1}{=}L_i^n$ if $i \le n$.

Swaption portfolio payoff

A portfolio of N_{opt} different swaptions with swap rates swap_n and maturity mat_n has payoff

$$P = \left\{ \prod_{i=0}^{N_{mat}-1} \frac{1}{1+\delta\,L_i} \right\} \left\{ \sum_{n=1}^{N_{opt}} 100\ (1-B_{\mathrm{mat}_n} - \mathrm{swap}_n S_{\mathrm{mat}_n})_+ \right\},$$

where

$$S_m = \sum_{i=1}^{m} \delta\,B_i,$$

and

$$B_m = \prod_{i=1}^{m} \frac{1}{1+\delta\,L_{N_{mat}+i-1}}.$$

Note that

$$1-B_m \;=\; \sum_{i=1}^{m} B_{i-1} - B_i \;=\; \sum_{i=1}^{m} \delta\,L_{N_{mat}+i-1}\,B_i,$$

and hence

$$1-B_m - \mathrm{swap}_n S_m = \sum_{i=1}^{m} \delta\,(L_{N_{mat}+i-1} - \mathrm{swap}_n)\,B_i,$$

making it clear that the swaption is concerned with a swap of the floating forward rate L_i and the fixed rate swap_n.