Monte Carlo Methods for Uncertainty Quantification: Assignment

The assignment is based on Lectures 1 & 2.

1. Suppose we want to evaluate the 5-dimensional integral

$$\int_{[0,1]^5} f(U)\, \mathrm{d}U$$

   where

$$f(U) = U_1 \left(\sin(U_2\, U_3) + \exp(U_4^2 + U_5^2)\right).$$

   Following the example in `basket_call.m` in Lecture 2, which is available from the course webpage, find the estimated values, and standard deviation of the error, when using a total of $N = 2^{18}, 2^{19}, 2^{20}$ sample points using

   - plain Monte Carlo;
   - Latin Hypercube with 128 strata in each dimension (so there are 128 points in each set) and $N/128$ independent sets of points so the total number of function evaluations is N;
   - Sobol points with 32 sets, so there are $N/32$ points in each set and a hence a total of N function evaluations.

2. Now suppose we are interested in estimating

$$\mathbb{E}\left[Z_1^2 \cos(Z_2+Z_3)\right]$$

   where Z_1, Z_2, Z_3 are independent Normal random variables with zero mean and unit variance.

   Modify your code to do this, using the same three approaches as in the first part.

Hand in your results as a single PDF file consisting of:

- your code
- your computed values and 3 standard deviation error estimates
- plots of error versus N
- comments on your results – do they agree with what you expected based on the lectures?