

Practical 3

For this practical you are to apply MLMC to the following two problems:

1. Consider the 1D equation

$$\frac{d}{dx} \left(\kappa \frac{dp}{dx} \right) = 0$$

on the unit interval $0 < x < 1$, subject to $p(0)=0, p(1)=1$, where $u(x) \equiv \log \kappa(x)$ is Gaussian with covariance $\mathbb{E}[u(x)u(y)] = \frac{1}{4} \exp(-|x-y|)$.

The objective is to estimate the quantity

$$\mathbb{E} \left[\int_0^1 \kappa \frac{dp}{dx} dx \right],$$

You can choose how to construct samples of u (Cholesky factorisation would be simplest), and how to approximate the 1D equation (e.g. finite difference or finite element method).

2. Given that Y and Z are independent log-normal random variables, with $\log Y \sim N(0, 1/2)$, $\log Z \sim N(0, 1/2)$, the objective is to estimate the nested expectation

$$\mathbb{E} \left[(\mathbb{E}[YZ | Y])^2 \right].$$

(Note: I chose a simple expression so that you can work out the exact value for comparison.)