Impact of a drop on a thin layer of fluid on a substrate.

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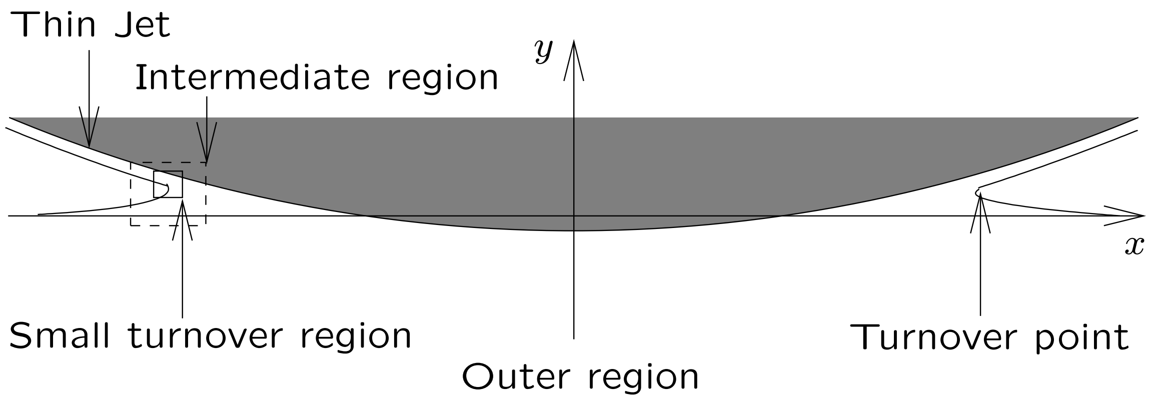
Classical Wagner theory

This deals with the rapid impact of a blunt body with small 'deadrise angle' on a half-space of liquid. Gravity, surface tension, viscosity and compressibility are all neglected. When the deadrise angle is small an asymptotic approach is possible: the flow is decomposed into regions which are linked by the technique of matched asymptotic expansions.



The inner (turnover region) is a standard Kelvin-Helmholtz flow.

The outer region sees the impact of an ‘effective flat plate’ extending between the turnover points. This is a linearised problem in which the kinematic and dynamic conditions are applied in linearised form on the undisturbed water level. The points corresponding to turnover are unknown.

$\phi = 0, \ \phi_y = h_t$ $\qquad\qquad$ $\phi_y = -1$ $\qquad\qquad$ $\phi = 0, \ \phi_y = h_t$

$-d(t)$ $\qquad\qquad$ $d(t)$

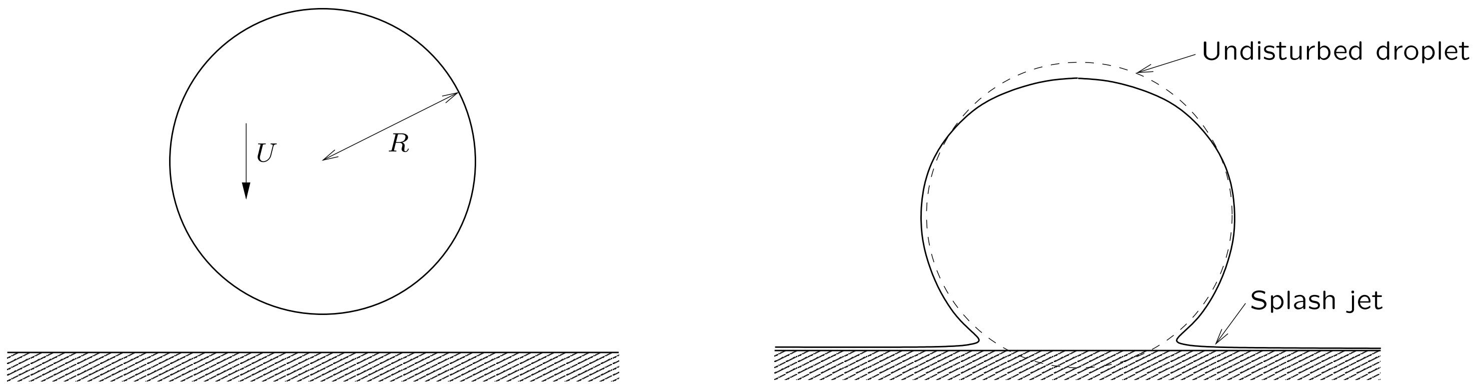
$$\phi_{xx} + \phi_{yy} = 0$$

The two are joined up by asymptotic matching, leading to the *Wagner condition*, that the free surface comes up to meet the impacting body at the turnover points. This is because the size of the turnover region is asymptotically small compared with the surface displacement.

In this way the ‘law of motion’ of the free points can be found.

There is also a thin splash jet which can be modelled by zero-gravity shallow water equations (NB: for all impactor shapes other than wedges the tip of the jet goes to infinity immediately!)

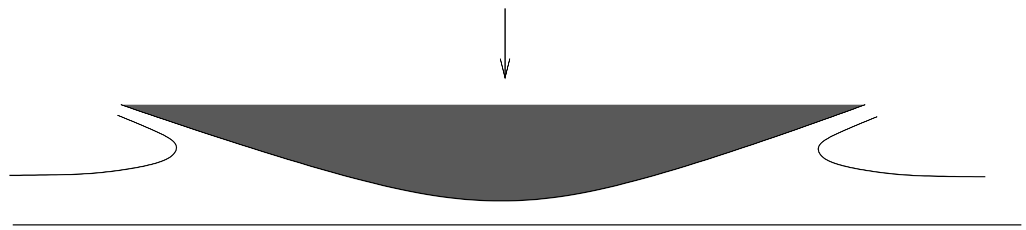
Wagner flows can be extended easily to symmetric impact of liquid bodies (the line of symmetry is like a rigid boundary). It can also be extended to impact of asymmetric bodies, and the impact of a liquid body on a flat substrate.



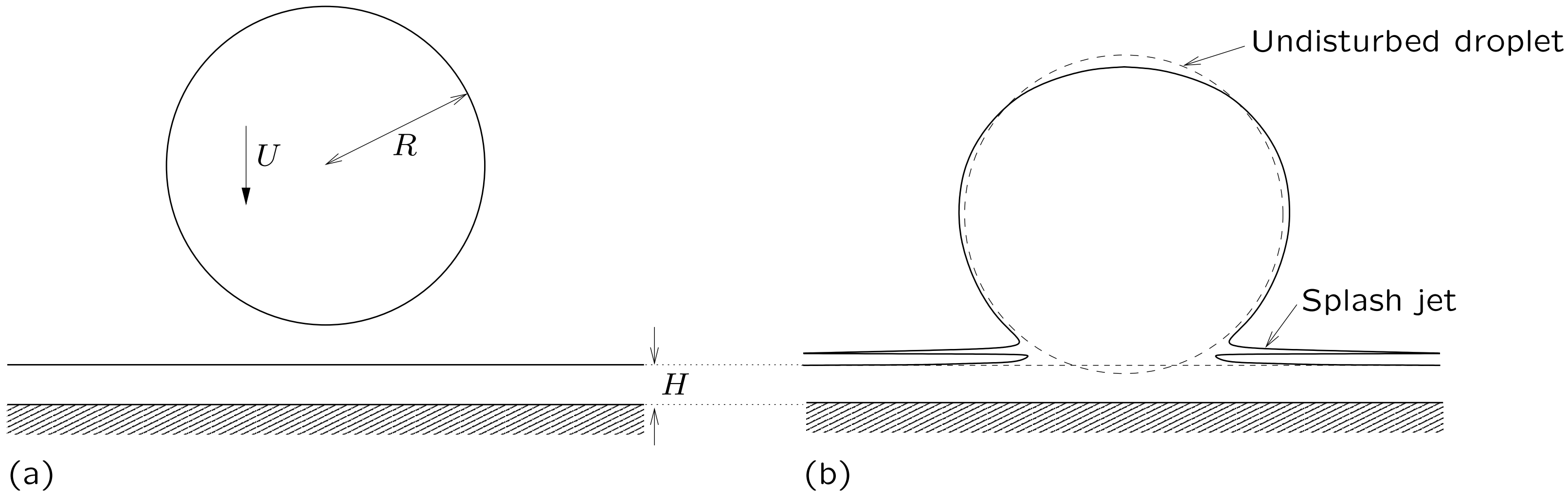
(a) (b)

Korobkin theory

This deals with the impact of a blunt body on a thin liquid layer above a substrate. The turnover regions are now of the same size as the layer depth and the flow under the body is approximated by an ‘inviscid squeeze film.



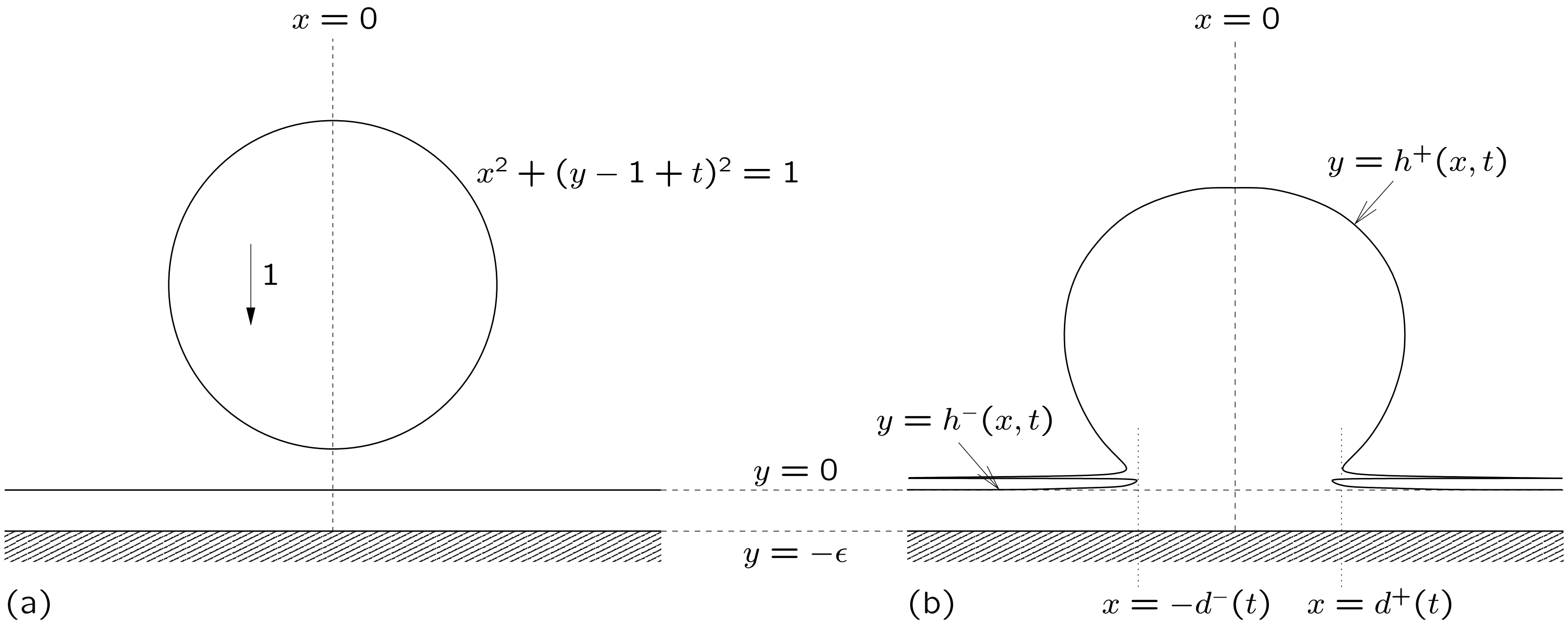
In this talk we deal with the impact of a liquid drop on a thin layer of liquid. The motivation is from models of icing of aircraft. The assumptions are as above: inviscid irrotational flow, no gravity, surface tension, viscosity or compressibility.



As in Wagner theory there are turnover points (now 4 of them), which form the jet root region.

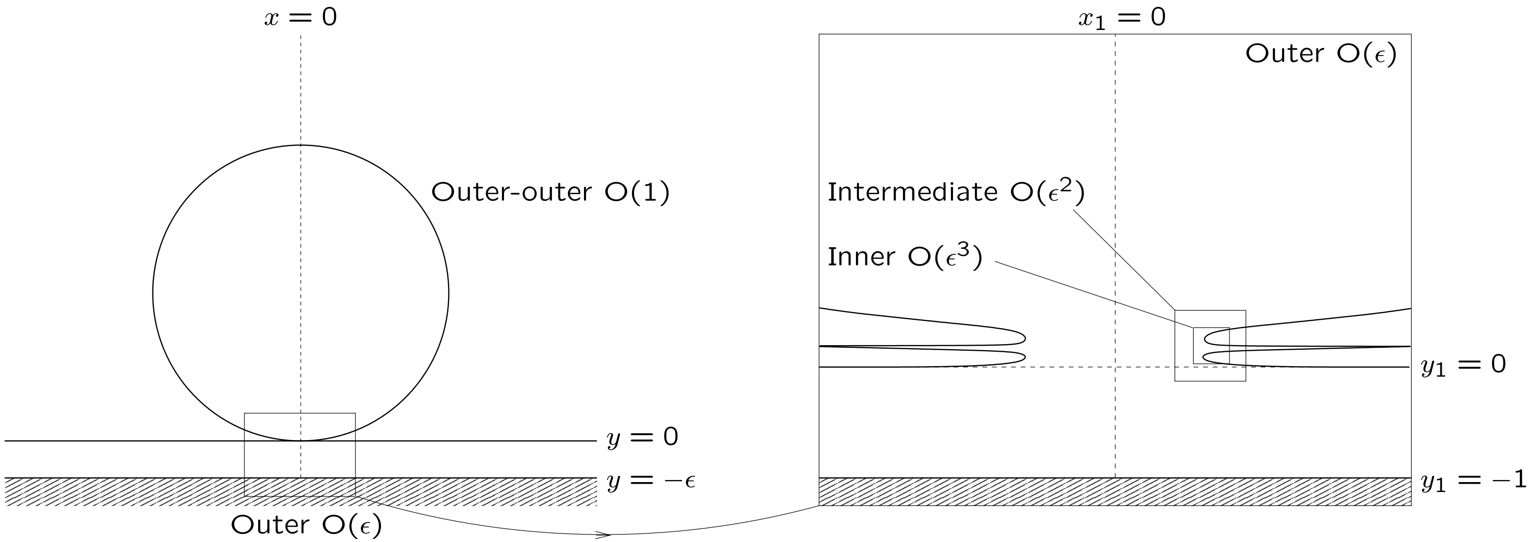
Let the impact speed be U, the droplet radius R and the layer depth H. We scale time with H/U, distnces with R.

We will assume that the parameter $\epsilon = H/R$ is small.

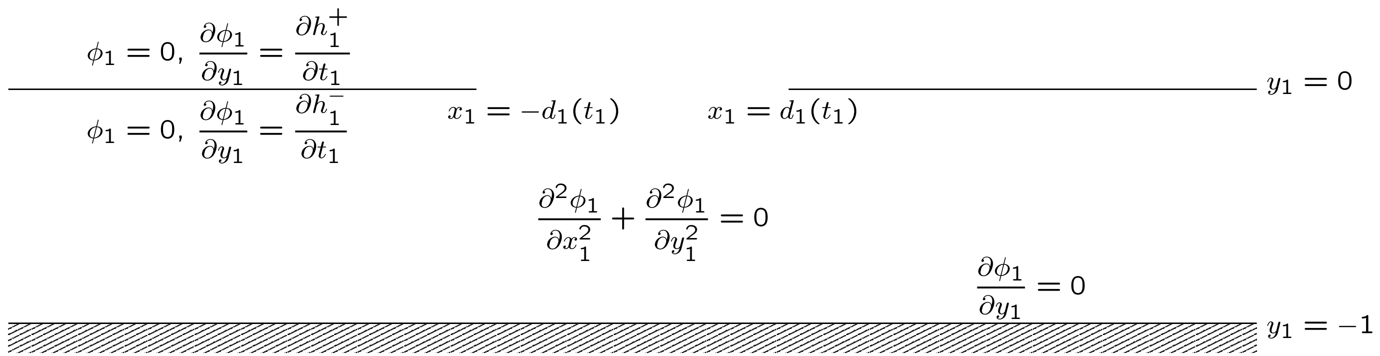


Initial stages: $t = O(\epsilon^2)$

In the first stages of impact, the 'nominal penetration' is very small. We can use standard Wagner theory with the addition of a base (which greatly increases the technical complications).

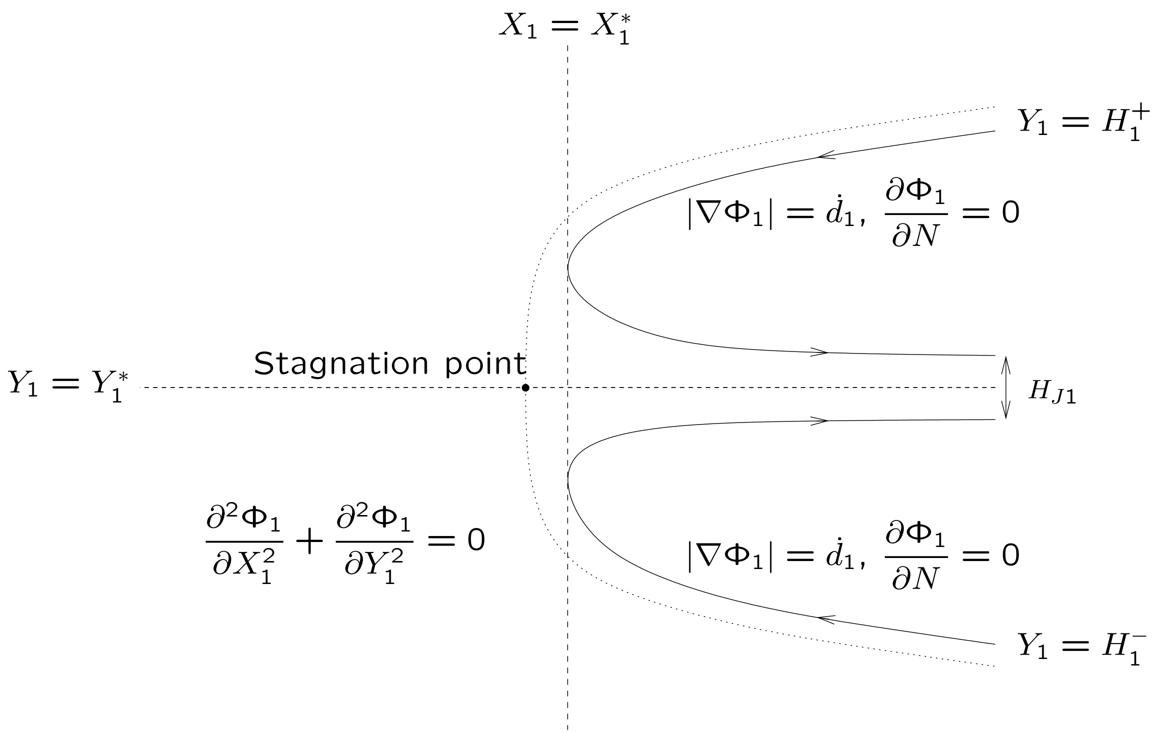


The outer flow is a Wagner-style impact of two liquid bodies.



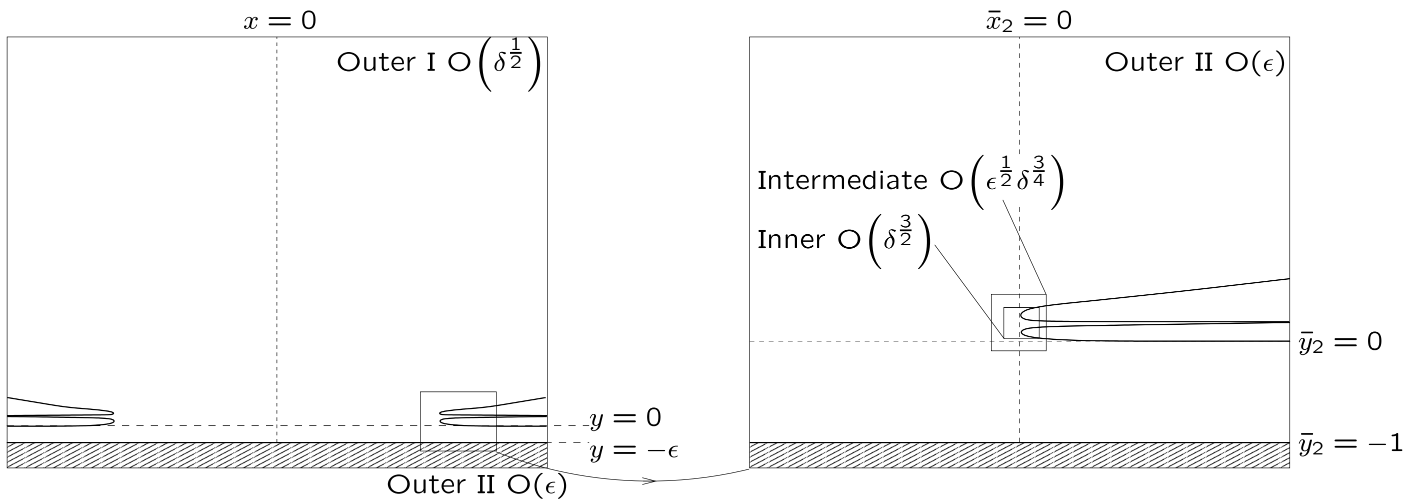
The ‘contact set’ between the jet roots initially grows as $t^{\frac{1}{2}}$. (This is the only part of the calculation that cannot easily be carried out for axisymmetric impact.)

The inner flow is a top-bottom symmetric jet-impact flow (in coordinates moving with the jet root) and at this stage the liquid going into the splash jet is 50% from the drop and 50% from the layer.



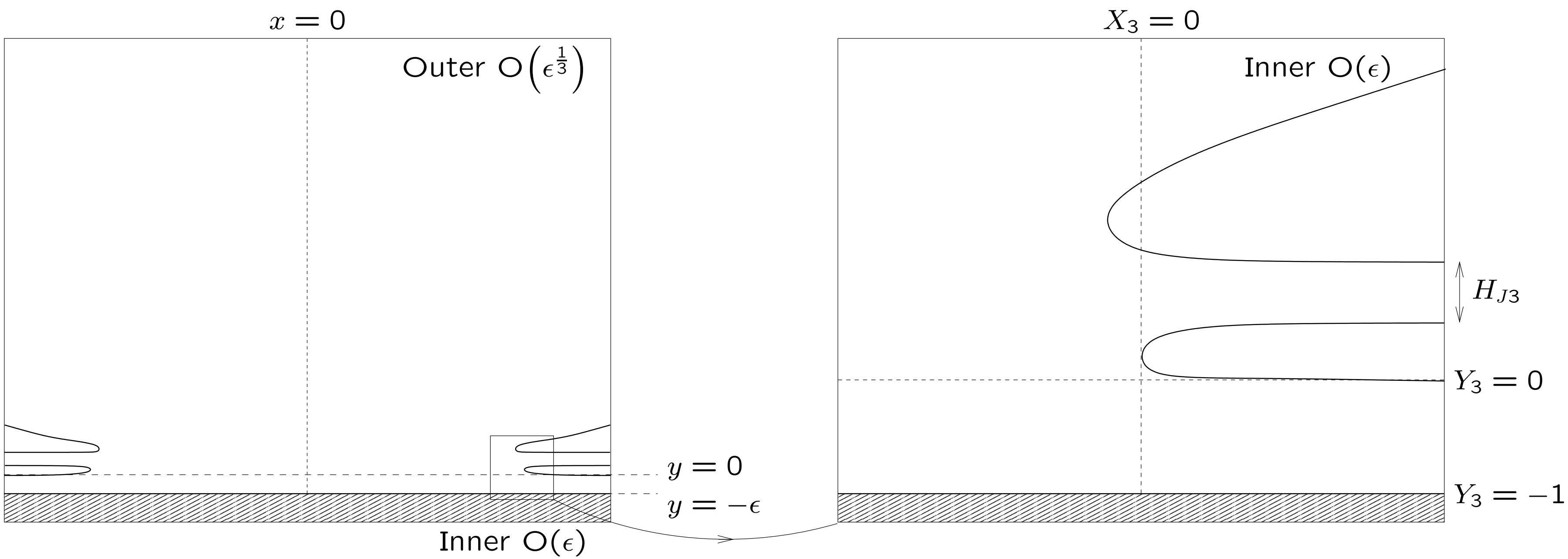
Intermediate stage: $\epsilon^2 \ll t \ll \epsilon^{\frac{2}{3}}$

Eventually the contact set grows until it is long compared with the layer depth. The turnover regions are still small and Wagner theory still applies. This stage is for $t = O(\delta)$ where $\epsilon^2 \ll \delta \ll \epsilon^{\frac{2}{3}}$. The fluid in the jet is still equally divided between the droplet and the layer. The outer problem is now a Wagner-style impact of a drop on a solid base.



Full interaction with the base: $t = O(\epsilon^{\frac{2}{3}})$.

The final stage that can be treated asymptotically is when the jet root is of the same size as the initial layer. This occurs when $t = O(\epsilon^{\frac{2}{3}})$. It is close in spirit to Korobkin flows.



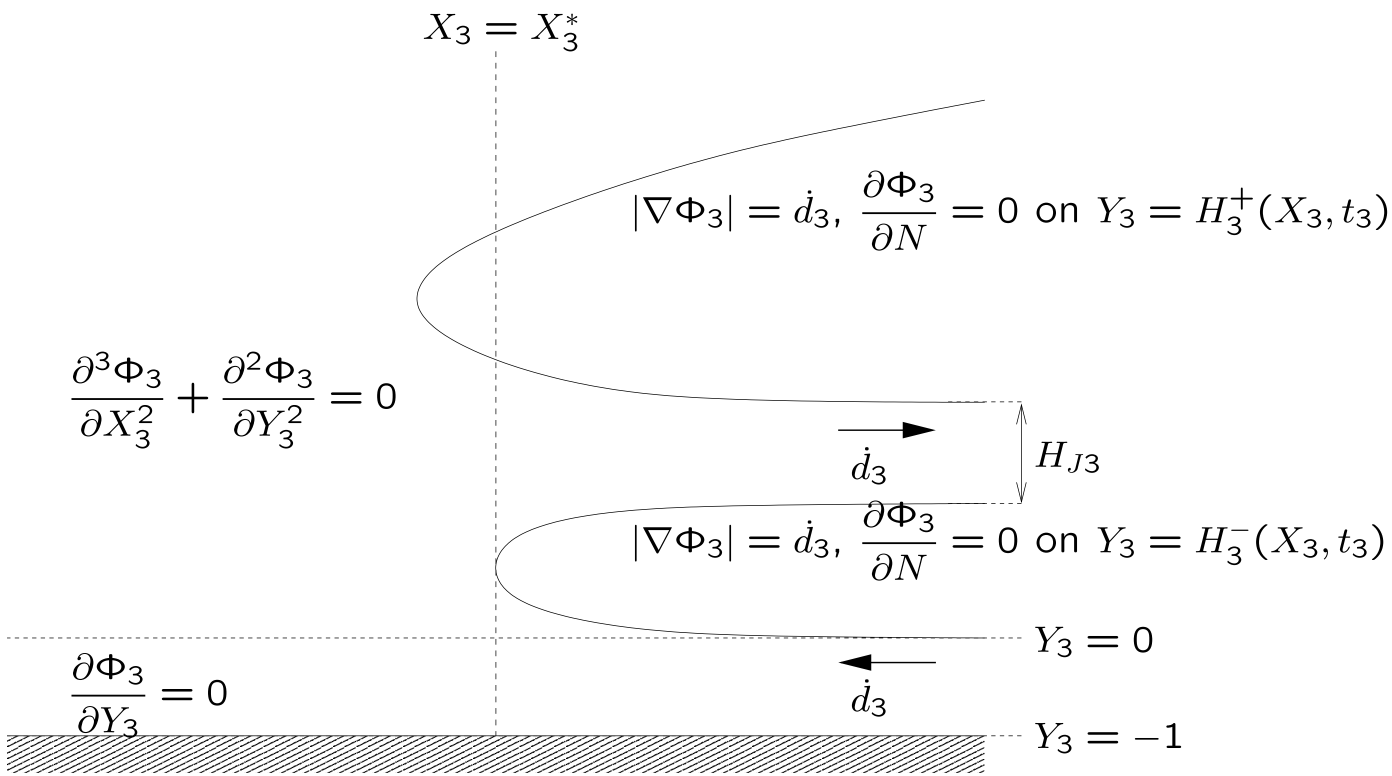
The outer problem is still a Wagner problem of a drop hitting a solid base. In the intermediate stage the drop switches from feeling a liquid layer to feeling only the base (to leading order) but the jet is still unaffected by the base.

Now the jet too feels the effect of the base.

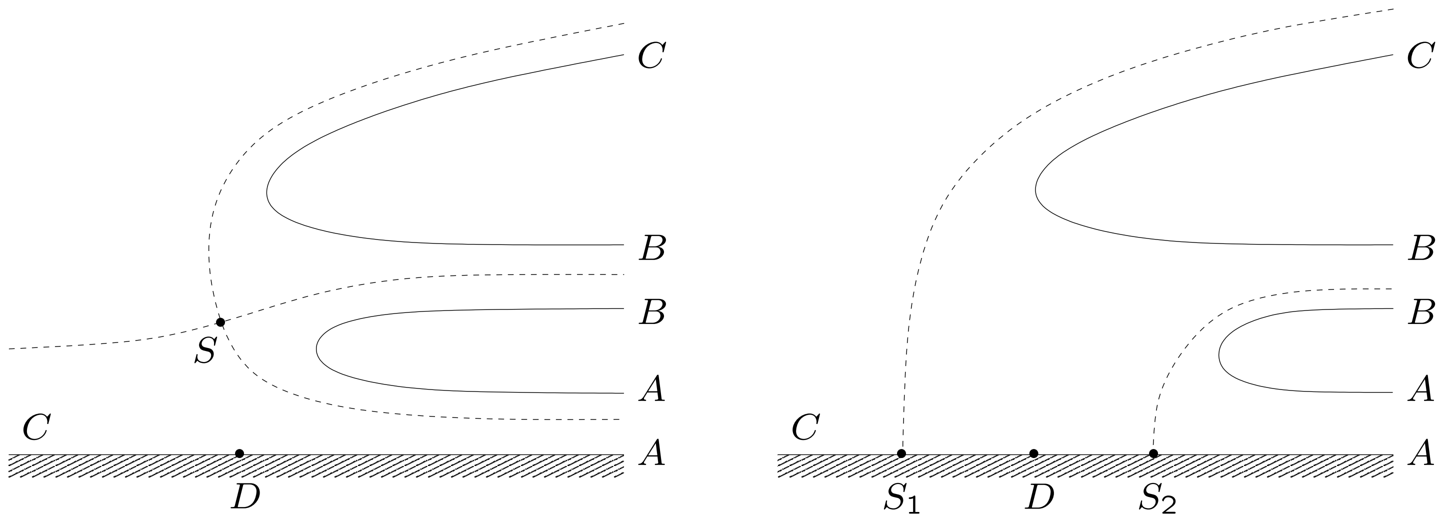
At the initiation of this stage (matching onto the end of the intermediate stage), the fluid entering the jet still comes equally from the drop and the layer.

What happens at time increases?

The inner flow is again a jet problem. There is a condition at infinity that comes from matching with the outer Wagner problem. Solve by conformal mapping.



The position of the stagnation point(s) is important. For small times there is one stagnation point, in the fluid. However at a finite time it *touches down* on the base. After this, there are two stagnation points on the wall.



If the drop is locally parabolic, with shape $x^2/2$ (as for a circular drop), touchdown occurs at dimensionless time

$$t = \left(\frac{16}{\pi}\right)^{\frac{2}{3}} \epsilon^{\frac{2}{3}} \quad \text{or, in dimensional terms,} \quad \left(\frac{16}{\pi}\right)^{\frac{2}{3}} \left(\frac{H}{R}\right)^{\frac{2}{3}} \frac{H}{U}.$$

Before touchdown, some of the fluid in the layer is left beneath the drop. After touchdown, it is all ejected. At touchdown, $\frac{1}{4}$ of the fluid in the jet comes from the layer.

Remarkably, even in this stage the jet thickness is given entirely by Wagner theory – that is, it is determined only by the drop radius and impact speed, not the layer thickness. Only the proportion of fluid in the jet from the drop is influenced by the layer thickness.

Summary and final remarks

We have shown how the impact of a drop on a thin layer of liquid on a solid base can be analysed to show the transition from liquid-liquid impact to liquid-solid impact. This transition occurs entirely in the early stages of impact, before the nominal penetration depth is comparable with the layer depth. It applies to axisymmetric drops as well as in two dimensions.

For times greater than $O(\epsilon^{\frac{2}{3}})$, the lower cavity (and right-hand stagnation point) in the jet root moves off much further than the upper one and the flow becomes more and more that of a drop hitting a dry wall. This can only be calculated numerically.