Professor Joyce T.C.C. Derived Algebraic Geometry Summer 2022

Miniprojects, for assessment of course during summer vacation

General guidance: *I am expecting you to produce an essay-type answer, of length perhaps 5 pages, ideally written in LaTeX and submitted as a PDF file by e-mail to* joyce@maths.ox.ac.uk, *by end September 2022. You could spend perhaps three days on the project, of which half might be reading references. If you were to go into some of these projects in depth then you would probably spend much longer. But I don't want you to waste your time on this; from the point of view of assessment, all you have to demonstrate is that you've learnt something and understood it, and can string a sentence together.*

These are just suggestions; feel free to choose your own topic.

Project 1. Write an essay on one of the following topics discussed in the lectures:

- What is derived geometry?
- What is an Artin stack?
- Triangulated categories and derived categories.
- Simplicial sets – what are they, and what are they used for?
- Model categories.
- ∞-categories.
- Derived schemes.
- Obstruction theories and virtual classes.
- Cotangent complexes.
- Choose your own topic.

Project 2. Discuss *orbifolds* in differential geometry, emphasizing the 2-categorical aspects.

Suggested references:
E. Lerman, *Orbifolds as stacks?*, arXiv:0806.4160.
I. Moerdijk, *Orbifolds as Groupoids: an Introduction*, math.DG/0203100.

Project 3. Find out and write about *moduli spaces* in Algebraic Geometry. Some topics you could consider:

- moduli of curves (Riemann surfaces);
- moduli of (algebraic) vector bundles (or more generally coherent sheaves), and Grothendieck's Quot scheme;
- Geometric Invariant Theory, and quotients in algebraic geometry.

Useful references:
F. Kirwan et al., *European women in mathematics: workshop on moduli spaces in mathematics and physics, Oxford, July 1998*, Hindawi, 2001, in particular, survey articles by F. Kirwan and R. Miro-Roig.
J. Harris and I. Morrison, *Moduli of curves*, Springer, 1998.
D. Gieseker, *Geometric invariant theory and applications to moduli problems*, pages 45-73 in Springer Lecture Notes in Math. 996, Springer, 1983.
D. Mumford, J. Fogarty and F. Kirwan, *Geometric invariant theory*, 3rd edition, Springer, 1994.

Project 4. Write about some aspect of *stability conditions* in Algebraic Geometry, especially in relation to coherent sheaves coh(X), or derived categories of coherent sheaves D^bcoh(X). Such stability conditions include *slope stability* for vector bundles or coherent sheaves on curves, *Gieseker stability* for coherent sheaves on higher-dimensional smooth schemes, and *Bridgeland stability* on derived categories D^bcoh(X). Why is stability important?