

Figure 1: Solutions of the (a) tip cell,  $n(x, y, t)$ , and (b) stalk cell,  $e(x, y, t)$ , densities given by the P-ABM and 2D snail-trail model subject to the TAF field  $c(x, y) = x$ , column averaged in the  $y$ -direction, at  $t = 0.2, 0.4, \dots, 2$ , with  $\kappa(x, y) = 1$ . Column averages were computed over the interval  $y \in [0.05, 0.95]$  in order to exclude possible edge effects. Key: P-ABM distribution (solid black lines); column averaged 2D snail-trail solution (red dashed lines). For colors, we refer to the online article. Initial conditions and parameter values: as in Figures 2 and 9 of the main text.

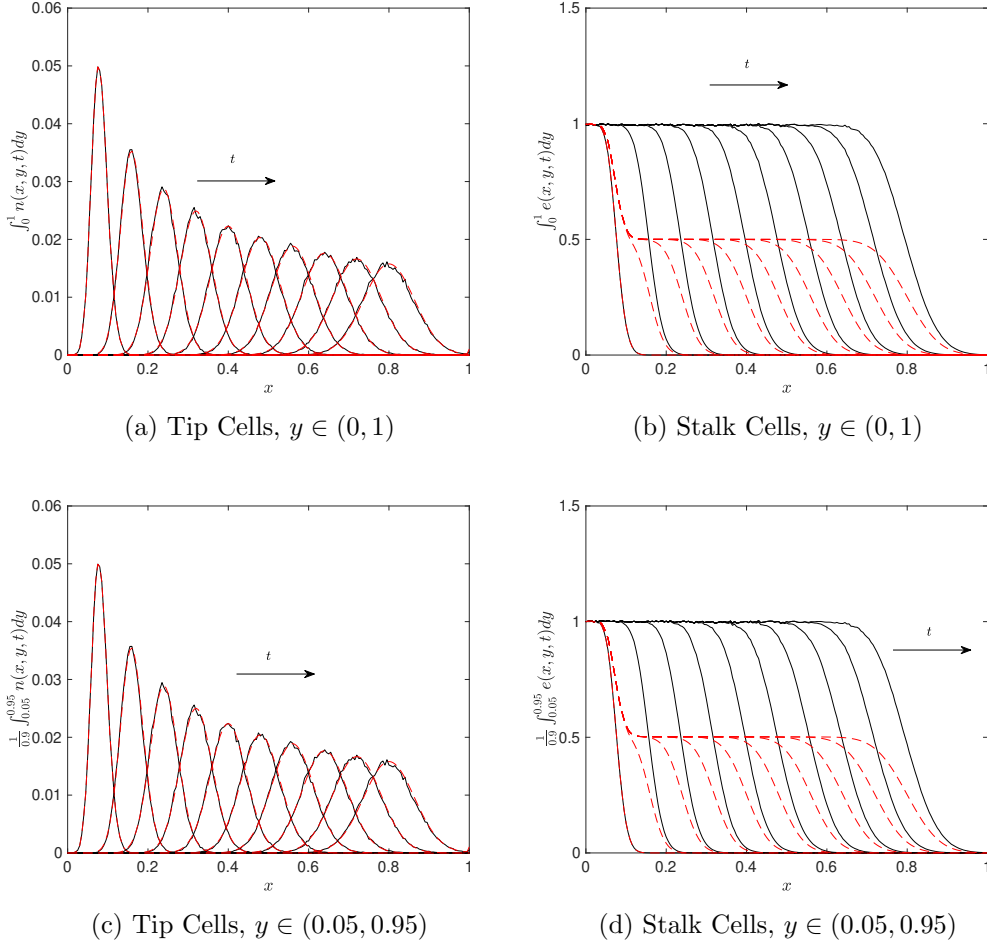


Figure 2: Solutions of the (a, c) tip cell,  $n(x, y, t)$ , and (b, d) stalk cell,  $e(x, y, t)$  densities given by the P-ABM and 2D snail-trail model, column averaged in the  $y$ -direction, at  $t = 0.2, 0.4, \dots, 2$ , with  $\kappa(x, y) = 1$  and  $\lambda = \beta_e = \beta_n = 0$ . Column averages were computed over the interval (a, b)  $y \in [0, 1]$  and (c, d)  $y \in [0.05, 0.95]$  in order to determine if edge effects caused differences between the two sets of results. Key: P-ABM distribution (solid black lines); column averaged 2D snail-trail solution (red dashed lines). The P-ABM was simulated with no anastomosis or branching allowed. The PDE was simulated on the interval  $t \in [0.2, 2]$  and initialized using the average P-ABM distribution at  $t = 0.2$  (see Appendix B of the main text for details). For colors, we refer to the online article.

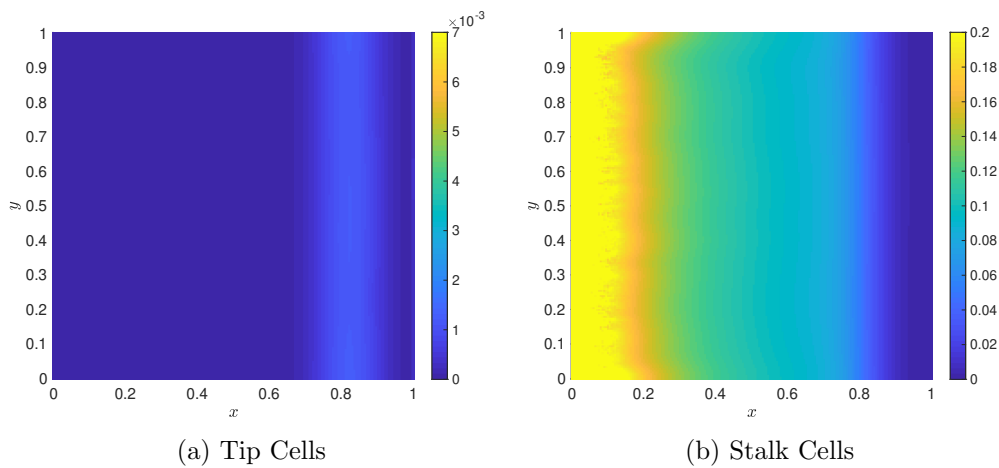
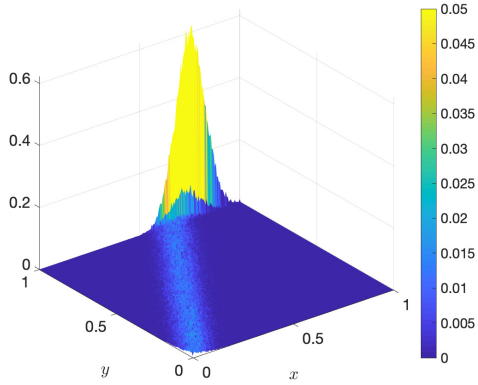
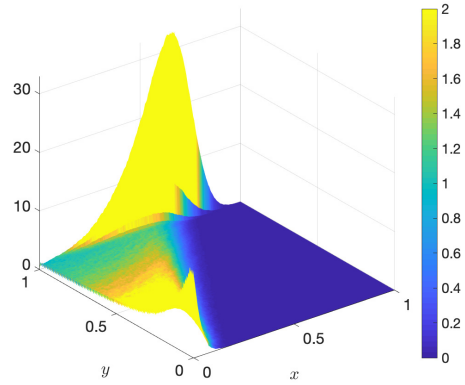


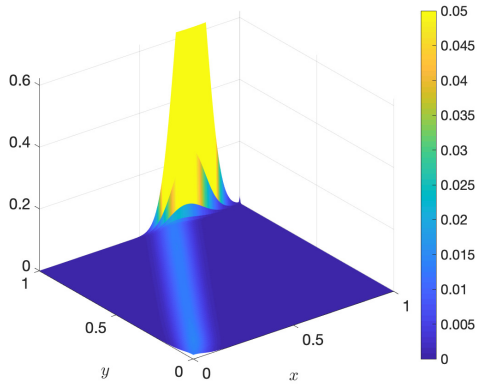
Figure 3: Heat map of (a) tip cell,  $n(x, y, t)$ , and (b) stalk cell,  $e(x, y, t)$  results given by the 2D snail-trail model at  $t = 2$ , subject to the TAF field  $c(x, y) = x$  with  $\kappa(x, y) = 2$ . The parameter  $\beta_e$  was fitted to the P-ABM results using the numerical methods described in Appendix B ( $\beta_e = 4.77$ , 95% CI:  $[4.73, 4.82]$ ). The snail-trail model was initialized at  $t = 0.2$  using the average P-ABM distribution at that time point.



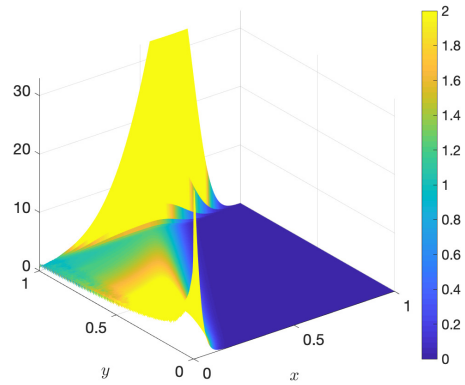
(a) P-ABM, Tip Cell Distribution



(b) P-ABM, Stalk Cell Distribution



(c) Snail-Trail PDE, Tip Cell Solution



(d) Snail-Trail PDE, Stalk Cell Solution

Figure 4: Surface plots of the 2D (a, c) tip cell,  $n(x, y, t)$ , and (b, d) stalk cell,  $e(x, y, t)$  densities given by the (a, b) P-ABM and (c, d) 2D snail-trail model at  $t = 2$ . Both models are subject to the TAF field  $c(x, y) = xy$  and neglect branching and anastomosis events (so that  $\lambda = \beta_e = \beta_n = 0$ ). Note that the snail-trail PDE appears to overestimate the P-ABM stalk cell distribution near  $(x, y) = (0, 0)$ ; this occurs because  $\kappa(x, y) \rightarrow \infty$  here. The continuous model over estimates the discrete solution near the edge  $y = 1$ , which is likely due to an edge effect. The PDE was simulated on the interval  $t \in [0.2, 2]$  and initialized using the average P-ABM distribution (see Appendix B of the main text for details). The P-ABM initial condition is described in Appendix A of the main text. For colors, we refer to the online article.

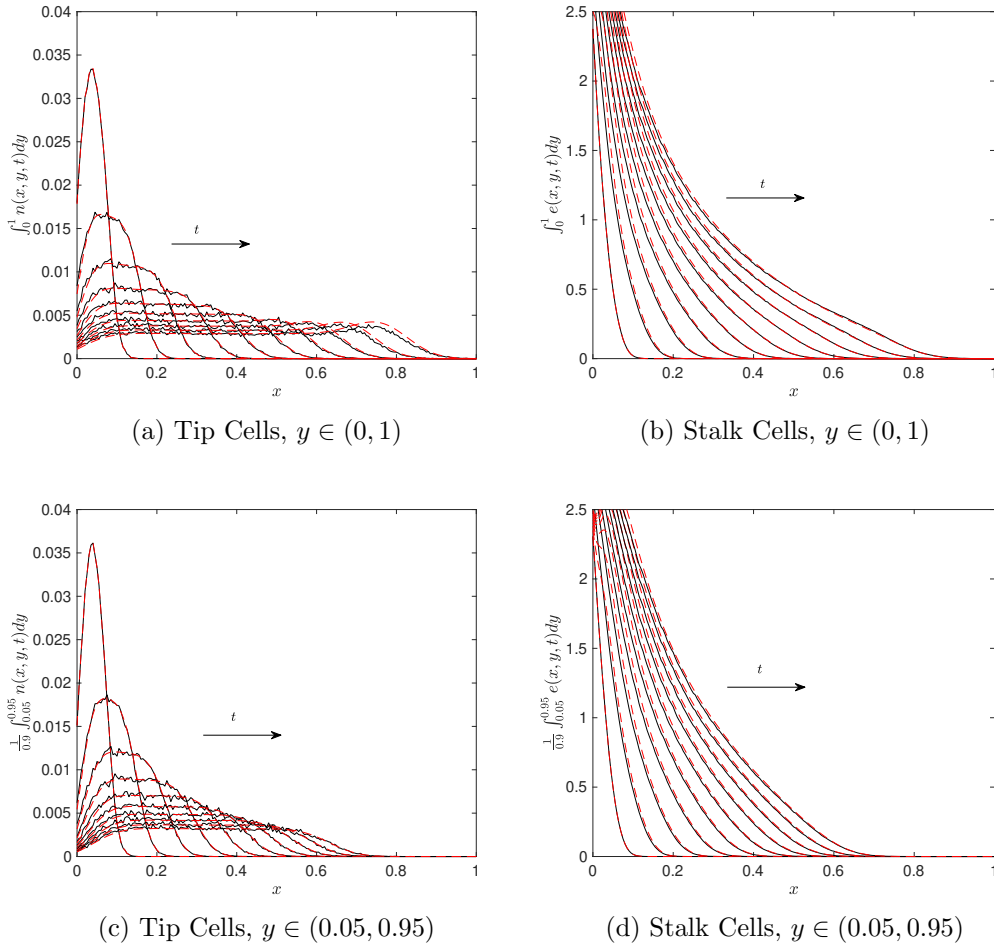


Figure 5: Solutions of the (a, c) tip cell,  $n(x, y, t)$ , and (b, d) stalk cell,  $e(x, y, t)$  densities given by the P-ABM and 2D snail-trail model subject to the TAF field  $c(x, y) = xy$ , column averaged in the  $y$ -direction, at  $t = 0.2, 0.4, \dots, 2$ , with  $\lambda = \beta_e = \beta_n = 0$ . Column averages were computed over the interval (a, b)  $y \in [0, 1]$  and (c, d)  $y \in [0.05, 0.95]$ , in order to determine if edge effects create differences between the two sets of results. The value of  $\kappa$  was taken to be constant and was computed using a nonlinear least squares fit to the P-ABM data ( $\kappa = 3.448$ , 95% CI:  $[3.445, 3.451]$ ; see Appendix B of the main text for details on the numerical methods). Key: P-ABM distribution (solid black lines); column averaged 2D snail-trail solution (red dashed lines). The PDE was simulated on the interval  $t \in [0.2, 2]$  and initialized using the average P-ABM distribution at  $t = 0.2$  (see Appendix B of the main text for details). For colors, we refer to the online article.

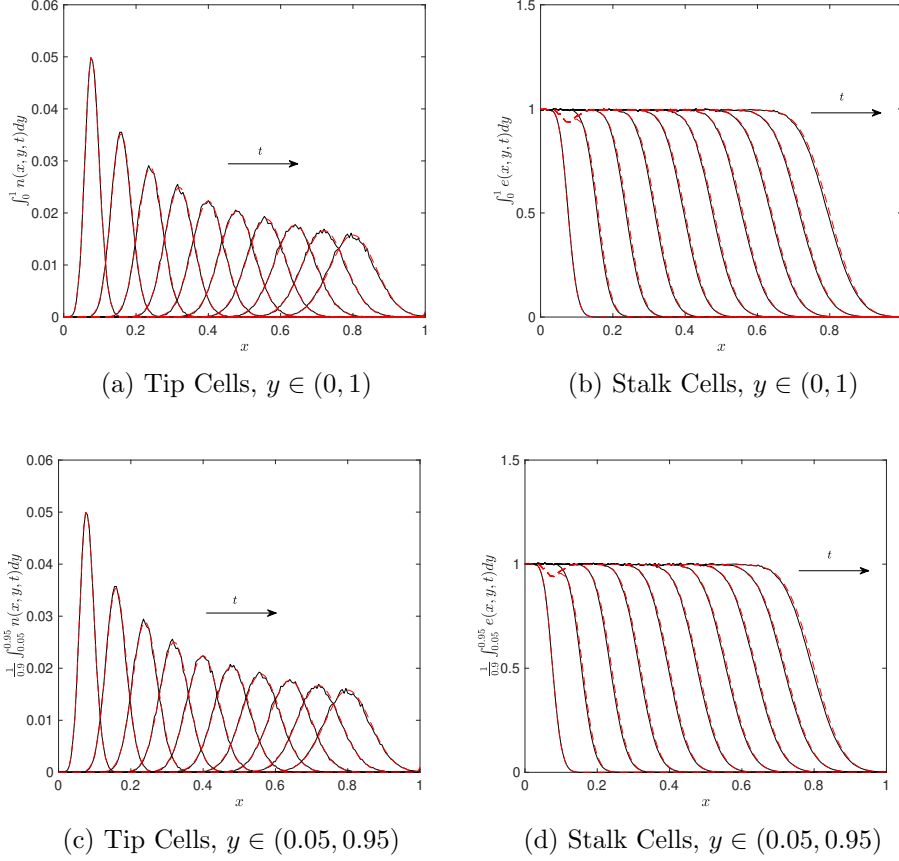


Figure 6: Solutions of the (a, c) tip cell,  $n(x, y, t)$ , and (b, d) stalk cell,  $e(x, y, t)$  densities given by the P-ABM and 2D snail-trail model subject to the TAF field  $c(x, y) = x$ , column averaged in the  $y$ -direction, at  $t = 0.2, 0.4, \dots, 2$ , with  $\lambda = \beta_e = \beta_n = 0$ . Column averages were computed over the interval (a, b)  $y \in [0, 1]$  or (c, d)  $y \in [0.05, 0.95]$ , in order to determine if there were edge effects. The value of  $\kappa$  was taken to be constant and was computed using a nonlinear least squares fit to the P-ABM data ( $\kappa = 1.9891$ , 95% CI:  $[1.9890, 1.9893]$ ; see Appendix B of the main text for details on the numerical methods). Key: P-ABM distribution (solid black lines); column averaged 2D snail-trail solution (red dashed lines). The computed value of  $\kappa$  is within 1% of the expected value of 2 predicted from equation (14) of the main text. The PDE was simulated on the interval  $t \in [0.2, 2]$  and initialized using the average P-ABM distribution at  $t = 0.2$  (see Appendix B of the main text for details). For colors, we refer to the online article.

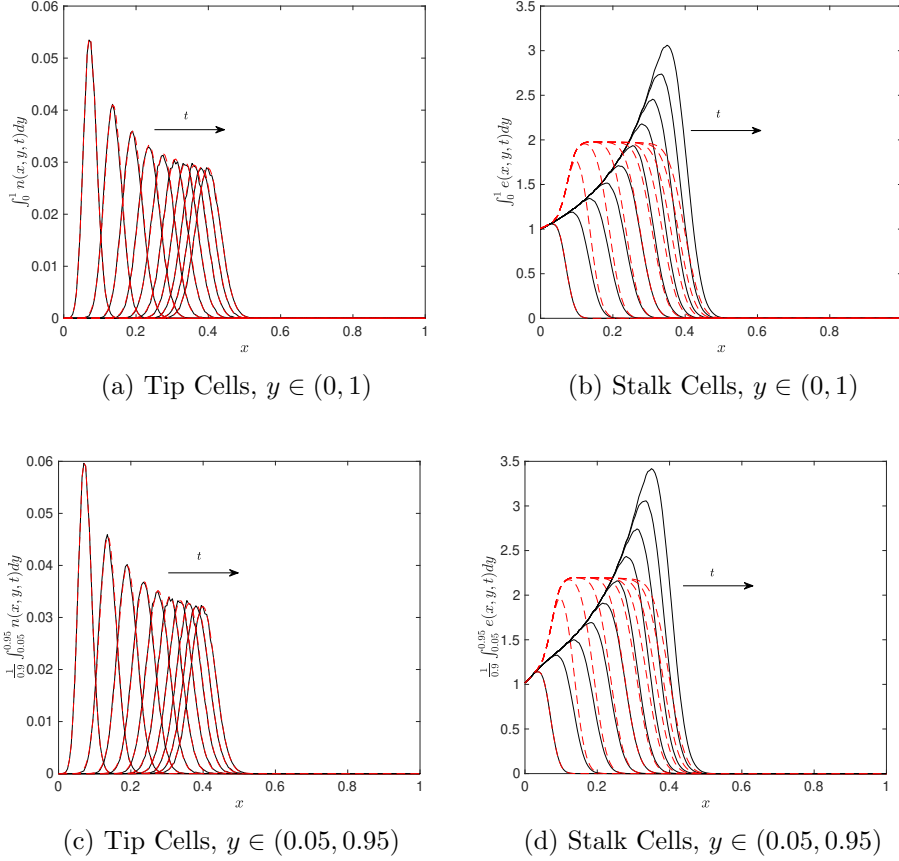


Figure 7: Solutions of the (a, c) tip cell,  $n(x, y, t)$ , and (b, d) stalk cell,  $e(x, y, t)$  densities given by the P-ABM and 2D snail-trail model subject to the TAF field  $c(x, y) = 1 - (x - \frac{1}{2})^2 - (y - \frac{1}{2})^2$ , column averaged in the  $y$ -direction, at  $t = 0.2, 0.4, \dots, 2$ , with  $\lambda = \beta_e = \beta_n = 0$ . Column averages were computed over the interval (a, b)  $y \in [0, 1]$  and (c, d)  $y \in [0.05, 0.95]$ , in order to determine if edge effects create differences between the two sets of results. The value of  $\kappa$  was taken to be constant and was computed using a nonlinear least squares fit to the P-ABM data ( $\kappa = 3.448$ , 95% CI:  $[3.445, 3.451]$ ; see Appendix B of the main text for details on the numerical methods). Key: P-ABM distribution (solid black lines); column averaged 2D snail-trail solution (red dashed lines). The PDE was simulated on the interval  $t \in [0.2, 2]$  and initialized using the average P-ABM distribution at  $t = 0.2$  (see Appendix B of the main text for details). For colors, we refer to the online article.

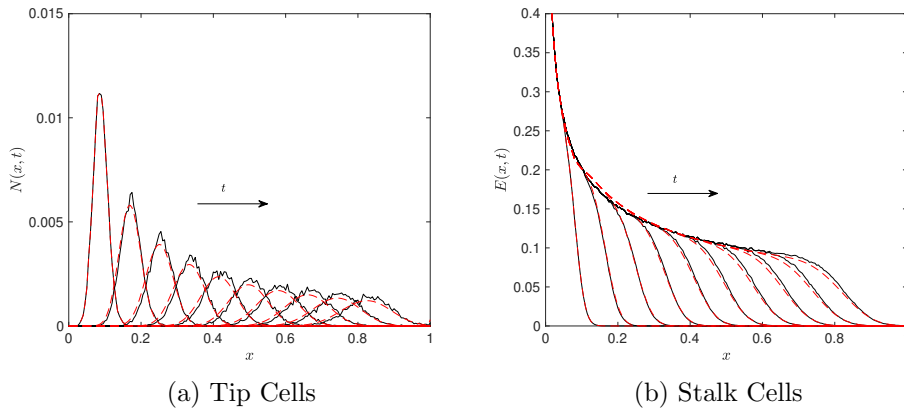


Figure 8: Solutions of the (a) tip cell,  $N(x, t)$ , and (b) stalk cell,  $E(x, t)$  densities given by the P-ABM and 1D snail-trail model at  $t = 0.2, 0.4, \dots, 2$ . The P-ABM results have been column averaged over the interval  $y \in [0, 1]$ , and are subject to the TAF field  $c(x, y) = x$ . The 1D snail-trail PDE is subject to the column averaged TAF field  $C(x) = x$  (so that  $\tilde{\kappa}(x) = 2$ ), and uses the parameter values listed in Table I of the main text. The value of the parameter  $\beta_e$  was fitted to the column averaged P-ABM data using a nonlinear least squares method ( $\beta_e = 4.61$ , 95% CI:  $[4.58, 4.64]$ ; see Appendix B of the main text for details on the numerical methods). Key: P-ABM distribution (solid black lines); 1D snail-trail solution (red dashed lines). The PDE was simulated on the interval  $t \in [0.2, 2]$  and initialized using the average P-ABM distribution at  $t = 0.2$  (see Appendix B of the main text for details). For colors, we refer to the online article.



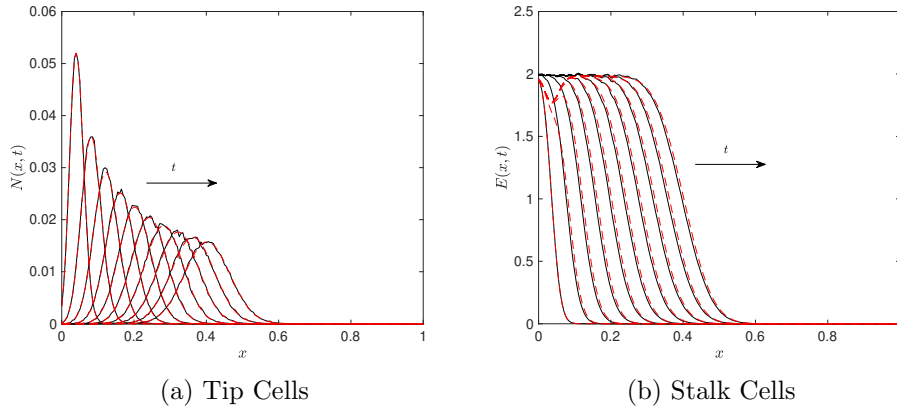


Figure 9: Solutions of the (a) tip cell,  $N(x, t)$ , and (b) stalk cell,  $E(x, t)$  densities given by the P-ABM and 1D snail-trail model at  $t = 0.2, 0.4, \dots, 2$ . The P-ABM results have been simulated without branching or anastomosis events, are column averaged over the interval  $y \in [0, 1]$ , and are subject to the TAF field  $c(x, y) = 0.5(x + y)$ . The 1D snail-trail PDE is subject to the column averaged TAF field  $C(x) = 0.5x + 0.25$ , with  $\lambda = \beta_e = \beta_n = 0$ . Key: P-ABM distribution (solid black lines); 1D snail-trail solution (red dashed lines). The PDE was simulated on the interval  $t \in [0.2, 2]$  and initialized using the average P-ABM distribution at  $t = 0.2$  (see Appendix B of the main text for details). For colors, we refer to the online article.

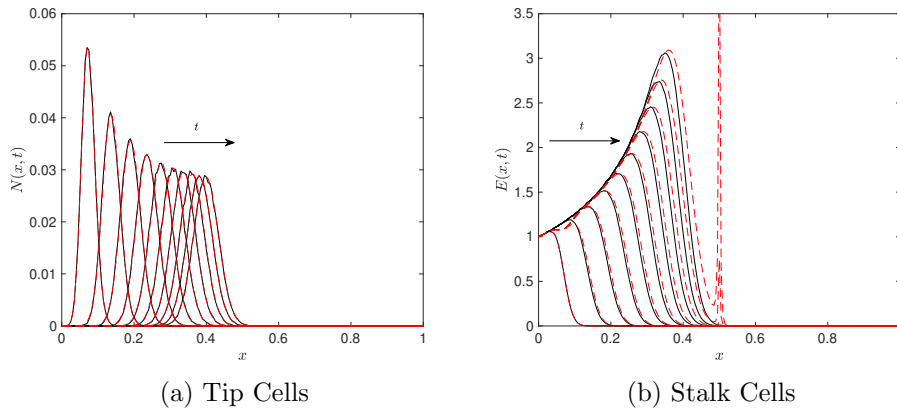


Figure 10: Solutions of the (a) tip cell,  $N(x, t)$ , and (b) stalk cell,  $E(x, t)$  densities given by the P-ABM and 1D snail-trail model at  $t = 0.2, 0.4, \dots, 2$ . The P-ABM results have been simulated without branching or anastomosis events, are column averaged over the interval  $y \in [0, 1]$ , and are subject to the TAF field  $c(x, y) = 1 - (x - 0.5)^2 - (y - 0.5)^2$ . The 1D snail-trail PDE is subject to the column averaged TAF field  $C(x) = 11/12 - (x - 1/2)^2$ , with  $\lambda = \beta_e = \beta_n = 0$ . Note the blow-up of the stalk cell solution near  $x = 1/2$ , which is where the TAF gradient is equal to 0; this occurs because the value of  $\tilde{\kappa}(x) \rightarrow \infty$  there. Key: P-ABM distribution (solid black lines); 1D snail-trail solution (red dashed lines). The PDE was simulated on the interval  $t \in [0.2, 2]$  and initialized using the average P-ABM distribution at  $t = 0.2$  (see Appendix B of the main text for details). For colors, we refer to the online article.