Interpolating actions for supersymmetric quantum field theory

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A new perturbative scheme has recently been applied to a supersymmetric quantum-field-theory model in which no conventional means for doing analytic calculations existed. We develop an alternative technique and find that it allows a very easy demonstration of the supersymmetric results: ground-state energy density $E=0$, and fermion-boson mass ratio $R=1$. Moreover, unlike other techniques, our method can be applied to models with spontaneous supersymmetry breaking, for which we illustrate the broken-supersymmetry results $E\neq 0$ and $R\neq 1$.

I. INTERPOLATING LAGRANGIANS

There has been much interest recently in *artificial* perturbative techniques for quantum field theories. The first of these techniques[1-6] involves writing a scalar interaction ϕ^{2p} as $\phi^{2(1+\delta)}$, and calculating the Green's functions as a power series in the parameter δ. This perturbation expansion yields a Lagrangian which is *logarithmic* in the fields, and we refer to it as the logarithmic expansion method (LEM). The latest technique[7,8] writes an interaction S as $\delta S+(1-\delta)S_0$, where S_0 is a free action, and the theory is again solved in powers of δ. Both techniques interpolate between a free theory at $\delta=0$, and the theory it is intended to solve ($\delta=1$), but the second technique relies on linear (as opposed to logarithmic) interpolation.

In two recent papers[9,10] the LEM was used to analyze a two-dimensional supersymmetric field theory, with the Euclidean-space Lagrangian

$$\mathcal{L}=\tfrac{1}{2}(\partial\phi)^2+\tfrac{1}{2}\bar{\psi}i\partial\!\!\!/\psi+\tfrac{1}{2}gS'(\phi)\bar{\psi}\psi+\tfrac{1}{2}g^2[S(\phi)]^2 , \tag{1.1}$$

where ψ is a Majorana spinor. The authors of Refs. 9 and 10 took $S(\phi)=(\phi^2)^{(1+\delta)/2}$, giving

$$\mathcal{L}=\tfrac{1}{2}(\partial\phi)^2+\tfrac{1}{2}\bar{\psi}i\partial\!\!\!/\psi+\tfrac{1}{2}g(1+\delta)(\phi^2)^{\delta/2}\bar{\psi}\psi +\tfrac{1}{2}g^2(\phi^2)^{(1+\delta)} . \tag{1.2}$$

It is known[9] that some models of this type exhibit spontaneous supersymmetry breaking, while others do not. For example, a $\phi^2\bar{\psi}\psi+\phi^6$ theory has an unbroken symmetry, while in a $\phi\bar{\psi}\psi+\phi^4$ theory the supersymmetry is spontaneously broken (as manifested by the ground-state energy being nonzero), because ϕ is not a positive operator. However, as explained in Ref. 9, setting $\delta=1$ in (1.2) yields a $|\phi|\bar{\psi}\psi+\phi^4$ interaction, rather than $\phi\bar{\psi}\psi+\phi^4$. One therefore expects the Lagrangian in (1.2) to exhibit *unbroken* supersymmetry for all values of δ. This was demonstrated in Refs. 9 and 10 by solving the theory in powers of δ.

In this paper we explore the possibility of using a *linear* interpolation in (1.1). For if we put

$$S(\phi)=\delta\phi^3-\lambda(1-\delta)\phi \tag{1.3}$$

in (1.1), where λ is an arbitrary dimensionless parameter, we obtain

$$\begin{aligned}\mathcal{L}=&\tfrac{1}{2}(\partial\phi)^2+\tfrac{1}{2}g^2\lambda^2\phi^2+\tfrac{1}{2}\bar{\psi}i\partial\!\!\!/\psi-\tfrac{1}{2}g\lambda\bar{\psi}\psi\\&+\tfrac{1}{2}\delta(-2+\delta)g^2\lambda^2\phi^2+\tfrac{1}{2}\delta g\lambda\bar{\psi}\psi\\&+\delta(-1+\delta)g^2\lambda\phi^4+\tfrac{1}{2}\delta^2g^2\phi^6+\tfrac{3}{2}\delta g\phi^2\bar{\psi}\psi .\end{aligned} \tag{1.4}$$

This Lagrangian can now be analyzed in powers of δ using conventional Feynman perturbation theory. We note that for $\delta=0$ we obtain a massive free theory, while for $\delta=1$ we obtain

$$\mathcal{L}=\tfrac{1}{2}(\partial\phi)^2+\tfrac{1}{2}\bar{\psi}i\partial\!\!\!/\psi+\tfrac{1}{2}g^2\phi^6+\tfrac{3}{2}g\phi^2\bar{\psi}\psi \tag{1.5}$$

which is the same as the Lagrangian in (1.2) with $\delta=2$. We therefore expect the theory in (1.4) to possess unbroken supersymmetry. This will be demonstrated in Sec. II, up to order δ^2.

We also see that the parameter λ is not present in (1.5). However, solving the theory in (1.4) as a δ series yields physical quantities with a nontrivial λ dependence, even for $\delta=1$. In Ref. 5 it is shown how to fix λ when a numerical result is required.

It was also pointed out in Ref. 9 that a perturbative expansion in powers of g of the models in (1.2) is not fruitful because it is plagued by infrared divergences, and that this was one reason for resorting to the complicated machinery of the LEM. We see from (1.4) that we *can* carry out a straightforward perturbative expansion in powers of δ, because of the mass terms that the linear interpolation (1.3) has introduced.

A further desirable feature of the linear interpolation approach is that we can extend it to include models in which the supersymmetry is spontaneously broken. If we put

$$S(\phi)=\delta\phi^2-\lambda(1-\delta)\phi \tag{1.6}$$

in (1.1), we obtain

$$\begin{aligned}\mathcal{L}=&\tfrac{1}{2}(\partial\phi)^2+\tfrac{1}{2}g^2\lambda^2\phi^2+\tfrac{1}{2}\bar{\psi}i\partial\!\!\!/\psi-\tfrac{1}{2}g\lambda\bar{\psi}\psi\\&+\tfrac{1}{2}\delta(-2+\delta)g^2\lambda^2\phi^2+\tfrac{1}{2}\delta g\lambda\bar{\psi}\psi\\&+\delta(-1+\delta)g^2\lambda\phi^3+\tfrac{1}{2}\delta^2g^2\phi^4+\delta g\phi\bar{\psi}\psi .\end{aligned} \tag{1.7}$$

The $\phi\bar{\psi}\psi$ term is a hint that this model will exhibit supersymmetry breaking, as manifested by the ground-state energy density being nonzero, and by the fermion and boson masses being unequal. We shall demonstrate how an expansion in δ can show this, up to order δ^2, in Sec. III.

II. A LINEAR INTERPOLATING LAGRANGIAN WITH UNBROKEN SUPERSYMMETRY

In this section we analyze the Lagrangian in (1.4) in powers of δ. The Feynman rules are shown in Fig. 1. We shall evaluate the ground-state energy density E, and the fermion-boson mass ratio R, to order δ^2, to show the supersymmetric results $E=0$ and $R=1$.

A. Calculation of the ground-state energy density E

The ground-state energy density E is given by minus the sum of the connected vacuum graphs. The Feynman graphs contributing to E at order δ are shown in Fig. 2. Evaluating each graph in Euclidean space yields

$$-3\delta g^2\lambda[\Delta(0)]^2 , \tag{2.1a}$$

$$-\delta g^2\lambda^2\Delta(0) , \tag{2.1b}$$

$$-\tfrac{1}{2}(-2\delta+\delta^2)g^2\lambda^2\Delta(0) , \tag{2.1c}$$

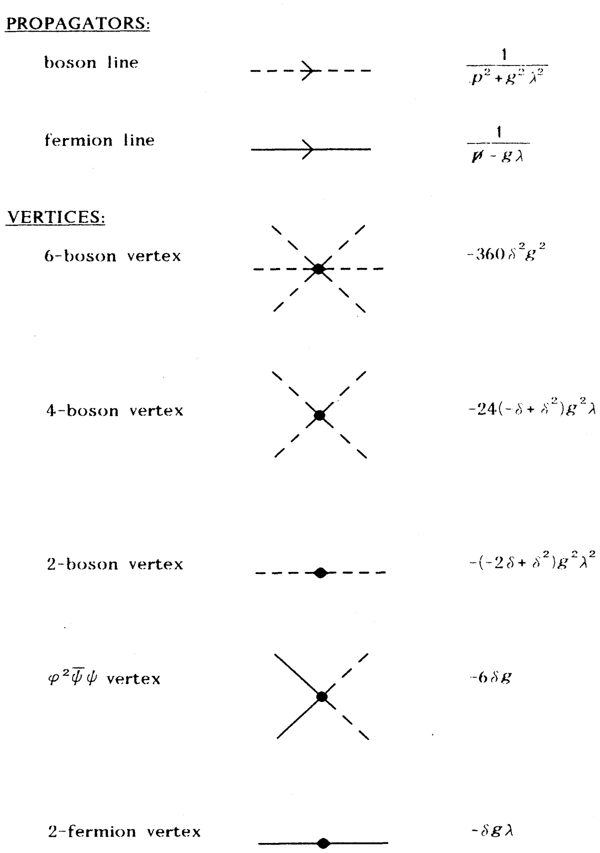

FIG. 1. Feynman rules for the supersymmetric Lagrangian $\mathcal{L}$ in (1.4).

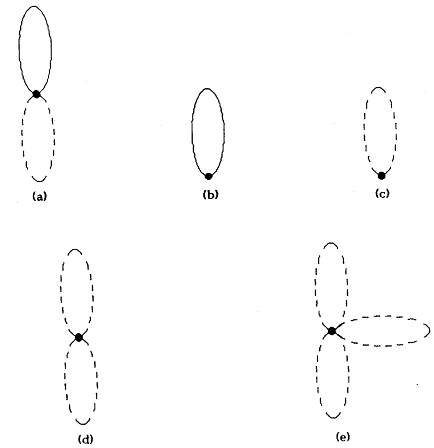

FIG. 2. Feynman diagrams which contribute, to first order in δ, to the ground-state energy density E of the theory in (1.4).

$$-3(-\delta+\delta^2)g^2\lambda[\Delta(0)]^2 , \tag{2.1d}$$

$$-\tfrac{15}{2}\delta^2 g^2[\Delta(0)]^3 , \tag{2.1e}$$

where $\Delta(0)$ represents the closed-loop boson propagator. In coordinate space the propagator for the boson is

$$\Delta(x)=(2\pi)^{-2}\int d^2p\frac{1}{p^2+g^2\lambda^2}\exp(i\mathbf{p}\cdot\mathbf{x}) \tag{2.2}$$

while the propagator for the fermion is

$$\Delta_F(x)=(2\pi)^{-2}\int d^2p\frac{1}{\not{p}-g\lambda}\exp(i\mathbf{p}\cdot\mathbf{x}) = (i\not{\partial}-g\lambda)\Delta(x) . \tag{2.3}$$

Summing (2.1a)–(2.1e) to order δ yields zero, implying $E=0$. Supersymmetry is thus unbroken to first order in δ.

To order δ^2, the graphs which contribute to E are those of Figs. 2 and 3. (There are more two-vertex graphs besides those shown in Fig. 3, but they are all of order δ^3 or higher.) The contribution of each diagram in Fig. 3 is as follows:

$$3\delta^2 g^2[\Delta(0)]^3-12\delta^2 g^4\lambda^2 I_4 , \tag{2.4a}$$

$$9\delta^2 g^4\lambda^2[\Delta(0)]^2 I_2 , \tag{2.4b}$$

$$\tfrac{9}{2}\delta^2 g^2[\Delta(0)]^3-9\delta^2 g^4\lambda^2[\Delta(0)]^2 I_2 , \tag{2.4c}$$

$$3\delta^2 g^2\lambda[\Delta(0)]^2-6\delta^2 g^4\lambda^3\Delta(0)I_2 , \tag{2.4d}$$

$$3(-2\delta^2+\delta^3)g^4\lambda^3\Delta(0)I_2 , \tag{2.4e}$$

$$36(-\delta^2+\delta^3)g^4\lambda^2[\Delta(0)]^2 I_2 , \tag{2.4f}$$

$$\tfrac{1}{2}\delta^2 g^2\lambda^2\Delta(0)-\delta^2 g^4\lambda^4 I_2 , \tag{2.4g}$$

$$\tfrac{1}{4}(4\delta^2-4\delta^3+\delta^4)g^4\lambda^4 I_2 , \tag{2.4h}$$

$$6(2\delta^2-3\delta^3+\delta^4)g^4\lambda^3\Delta(0)I_2\ , \tag{2.4i}$$

$$12(\delta^2-2\delta^3+\delta^4)g^4\lambda^2 I_4\ , \tag{2.4j}$$

$$36(\delta^2-2\delta^3+\delta^4)g^4\lambda^2[\Delta(0)]^2 I_2\ , \tag{2.4k}$$

where

$$I_k=\int d^2x[\Delta(x)]^k\ . \tag{2.4l}$$

To obtain the expressions in (2.4) in the form shown, we made use of the identity[10]

$$\mathrm{Tr}\int d^2x[\Delta(x)]^n\Delta_F(x)\bar{\Delta}_F(x)\exp(-i\mathbf{p}\cdot\mathbf{x})=-\frac{2}{n+1}[\Delta(0)]^{n+1}+\frac{2}{n+1}\left[(n+2)g^2\lambda^2+\frac{p^2}{n+2}\right]\int d^2x[\Delta(x)]^{n+2}\exp(-i\mathbf{p}\cdot\mathbf{x})\ . \tag{2.5}$$

Summing (2.1a)–(2.1e) with (2.4a)–(2.4k) to order δ^2 gives zero, so that supersymmetry remains unbroken through second order in δ.

B. Calculation of the Fermion-boson mass ratio R

To evaluate the boson or fermion mass of the theory in (1.4) we must calculate the one-particle-irreducible Green's functions with two external boson or two external fermion lines, respectively (i.e., the boson and fermion self-energies).

The full boson propagator $D(p)$ is given by

$$D(p)=\frac{1}{p^2+g^2\lambda^2-\Pi(p)}\ , \tag{2.6}$$

where $\Pi(p)$ is the boson self-energy. The full fermion propagator $S(p)$ is given by

$$S(p)=\frac{1}{\not p-[g\lambda+\Sigma(p)]}\ , \tag{2.7}$$

where $\Sigma(p)$ is the fermion self-energy.

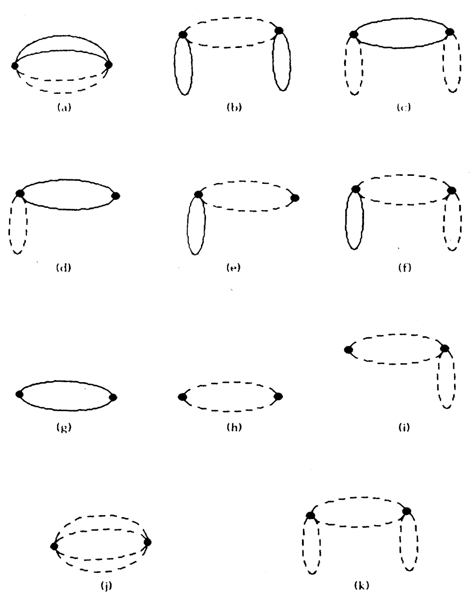

FIG. 3. Graphs which contribute to E in second order, for the theory in (1.4).

To order δ, the Feynman diagrams which contribute to $\Pi(p)$ are shown in Fig. 4. The contribution from each of these graphs is

$$-(-2\delta+\delta^2)g^2\lambda^2\ , \tag{2.8a}$$

$$-12(-\delta+\delta^2)g^2\lambda\Delta(0)\ , \tag{2.8b}$$

$$-45\delta^2g^2[\Delta(0)]^2\ , \tag{2.8c}$$

$$-6\delta g^2\lambda\Delta(0)\ . \tag{2.8d}$$

Summing (2.8a)–(2.8d) to order δ yields

$$\Pi(p)_{O(\delta)}=2\delta g^2\lambda^2+6\delta g^2\lambda\Delta(0)\ . \tag{2.9}$$

Then, using (2.6), we deduce that to order δ the mass squared of the boson is

$$m_b^2=g^2\lambda^2-2\delta g^2\lambda^2-6\delta g^2\lambda\Delta(0)\ . \tag{2.10}$$

The Feynman graphs which contribute in first order to the fermion self-energy $\Sigma(p)$ are shown in Fig. 5. We evaluate them to give

$$\Sigma(p)_{O(\delta)}=-\delta g\lambda-3\delta g\Delta(0)\ . \tag{2.11}$$

Then (2.7) implies that, to order δ, the fermion mass is

$$m_f=g\lambda-\delta g\lambda-3\delta g\Delta(0)\ . \tag{2.12}$$

Comparing (2.10) and (2.12), we see that, to order δ, the ratio

$$R=\frac{m_f}{m_b}=1 \tag{2.13}$$

which confirms the supposition that (1.4) has unbroken

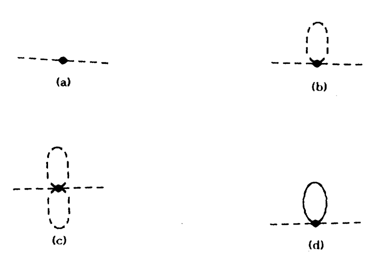

FIG. 4. Graphs which contribute to the boson self-energy to first order in δ, for the theory in (1.4).

FIG. 5. Graphs which contribute to the fermion self-energy to first order in δ, for the theory in (1.4).

supersymmetry to first order in δ.

To order δ^2, the graphs which contribute to the boson two-point function are those in Figs. 4 and 6. The contributions from the diagrams in Fig. 6 are

$$18\delta^2 g^2[\Delta(0)]^2-(54\delta^2 g^4\lambda^2+6\delta^2 g^2 p^2)I_{3,p} \ , \tag{2.14a}$$

$$18\delta^2 g^2[\Delta(0)]^2-36\delta^2 g^4\lambda^2\Delta(0)I_2 \ , \tag{2.14b}$$

$$6\delta^2 g^2\lambda\Delta(0)-12\delta^2 g^4\lambda^3 I_2 \ , \tag{2.14c}$$

$$72(-\delta^2+\delta^3)g^4\lambda^2\Delta(0)I_2 \ , \tag{2.14d}$$

$$12(2\delta^2-3\delta^3+\delta^4)g^4\lambda^3 I_2 \ , \tag{2.14e}$$

$$96(\delta^2-2\delta^3+\delta^4)g^4\lambda^2 I_{3,p} \ , \tag{2.14f}$$

$$144(\delta^2-2\delta^3+\delta^4)g^4\lambda^2\Delta(0)I_2 \ , \tag{2.14g}$$

where

$$I_{k,p}=\int d^2x[\Delta(x)]^k\exp(-i\mathbf{p}\cdot\mathbf{x}) \ . \tag{2.14h}$$

Summing (2.8a)–(2.8d) with (2.14a)–(2.14g) to order δ^2 gives $\Pi(p)$. If we write

$$\Pi(p)=A(p^2)p^2+B(p^2) \ , \tag{2.15}$$

where the explicit p^2 factor appears in (2.14a), then to order δ^2 the mass squared of the boson is given by the zero of

$$p^2+g^2\lambda^2-B(p^2)+g^2\lambda^2 A(p^2) \ . \tag{2.16}$$

Evaluating (2.16) to order δ^2 reveals that the mass squared of the boson is given by the zero of

$$p^2+g^2\lambda^2-(2\delta-\delta^2)g^2\lambda^2-6(\delta-\delta^2)g^2\lambda\Delta(0) +9\delta^2 g^2[\Delta(0)]^2-12\delta^2 g^4\lambda^3 I_2-36\delta^2 g^4\lambda^2\Delta(0)I_2 -48\delta^2 g^4\lambda^2 I_{3,p} \ . \tag{2.17}$$

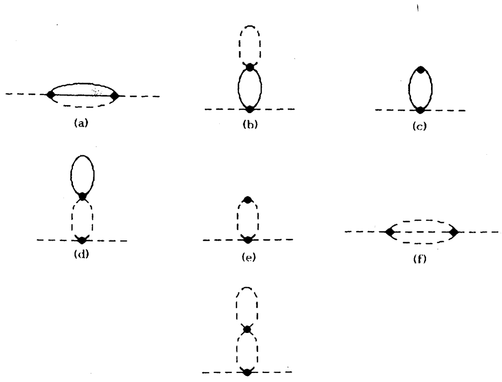

FIG. 6. Two-vertex graphs contributing to the boson self-energy for the theory in (1.4).

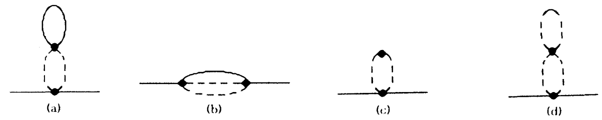

FIG. 7. Two-vertex graphs contributing to the fermion self-energy to order δ^2, for the theory in (1.4).

To evaluate the fermion self-energy $\Sigma(p)$ to order δ^2 we use the Feynman diagrams in Figs. 5 and 7. The contributions from the graphs in Fig. 7 are

$$18\delta^2 g^3\lambda\Delta(0)I_2 \ , \tag{2.18a}$$

$$-18\delta^2 g^3\lambda I_{3,p}-6\delta^2 g^2\not{p}I_{3,p} \ , \tag{2.18b}$$

$$3(-2\delta^2+\delta^3)g^3\lambda^2 I_2 \ , \tag{2.18c}$$

$$36(-\delta^2+\delta^3)g^3\lambda\Delta(0)I_2 \ . \tag{2.18d}$$

Adding (2.11) to (2.18a)–(2.18d), evaluated to order δ^2, gives $\Sigma(p)$. Writing

$$\Sigma(p)=a(p^2)\not{p}+b(p^2) \ , \tag{2.19}$$

then using (2.7) and the fact that $\not{p}\not{p}=-p^2$ in Euclidean space, we find that to order δ^2 the mass squared of the fermion is given by the zero of

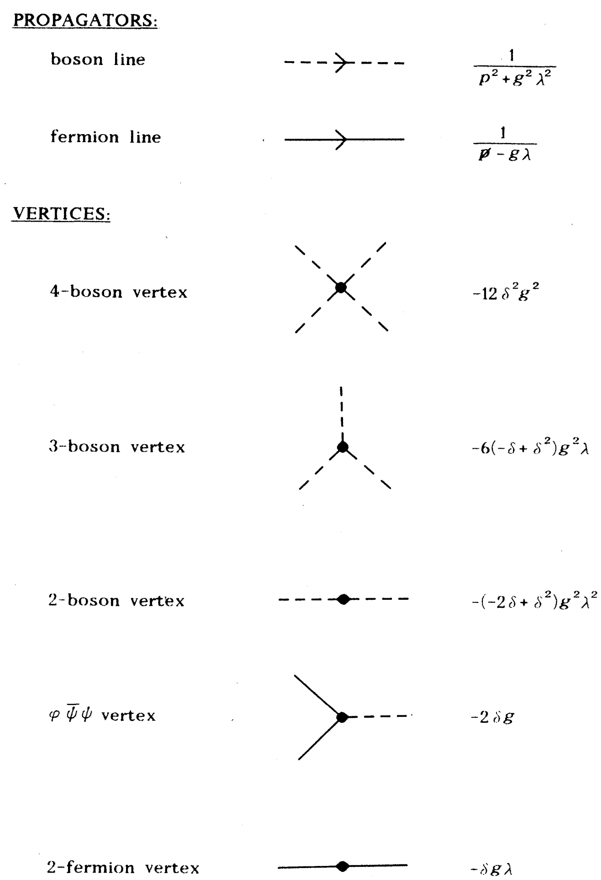

FIG. 8. Feynman rules for Lagrangian in (1.7).

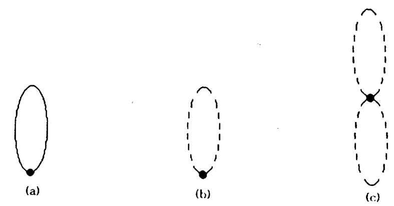

FIG. 9. Diagrams contributing to the ground-state energy density E of the theory in (1.7), to first order in δ.

$$p^2+g^2\lambda^2+2g\lambda b(p^2)+[b(p^2)]^2+2g^2\lambda^2 a(p^2)\ . \tag{2.20}$$

On evaluating (2.20) we recover (2.17), so that

$$R=\frac{m_f}{m_b}=1 \tag{2.21}$$

to order δ^2 for the theory in (1.4). We conclude, therefore, that the δ expansion allows a categorical demonstration of unbroken supersymmetry for the theory in (1.4).

III. AN INTERPOLATING ACTION WITH BROKEN SUPERSYMMETRY

A. Calculation of the ground-state energy density E

The Feynman rules for the Lagrangian in (1.7) are shown in Fig. 8. We evaluate the diagrams in Fig. 9, which contribute to the ground-state energy density E to order δ. The graphs of Fig. 9 yield

$$-\delta g^2\lambda^2\Delta(0)\ , \tag{3.1a}$$

$$-\tfrac{1}{2}(-2\delta+\delta^2)g^2\lambda^2\Delta(0)\ , \tag{3.1b}$$

$$-\tfrac{3}{2}\delta^2 g^2[\Delta(0)]^2\ . \tag{3.1c}$$

Summing (3.1a)–(3.1c) to order δ yields zero, so we must conclude that supersymmetry is unbroken to first order in δ. This result can be traced to the fact that only the two-boson and two-fermion vertices contribute to E at order δ, so that we are effectively dealing with the Lagrangian

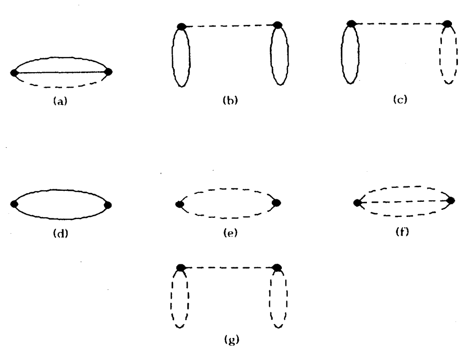

FIG. 10. Graphs which contribute to E in second order, for the theory in (1.7).

FIG. 11. Feynman graphs contributing to the order-δ boson self-energy of the theory in (1.7).

$$\mathcal{L}=\tfrac{1}{2}(\partial\phi)^2+\tfrac{1}{2}(1-2\delta)g^2\lambda^2\phi^2+\tfrac{1}{2}\bar{\psi}i\partial\!\!\!/\psi-\tfrac{1}{2}(1-\delta)g\lambda\bar{\psi}\psi \tag{3.2}$$

which is manifestly supersymmetric. We must proceed to higher orders in δ to see if symmetry breaking occurs.

To evaluate E to order δ^2 we add the diagrams in Fig. 10 to those in Fig. 9. The contributions of the graphs in Fig. 10 are

$$\delta^2 g^2[\Delta(0)]^2-3\delta^2 g^4\lambda^2 I_3\ , \tag{3.3a}$$

$$2\delta^2 g^4\lambda^2[\Delta(0)]^2 I_1\ , \tag{3.3b}$$

$$6(-\delta^2+\delta^3)g^4\lambda^2[\Delta(0)]^2 I_1\ , \tag{3.3c}$$

$$\tfrac{1}{2}\delta^2 g^2\lambda\Delta(0)-\delta^2 g^4\lambda^4 I_2\ , \tag{3.3d}$$

$$\tfrac{1}{4}(4\delta^2-4\delta^3+\delta^4)g^4\lambda^4 I_2\ , \tag{3.3e}$$

$$3(\delta^2-2\delta^3+\delta^4)g^4\lambda^2 I_3\ , \tag{3.3f}$$

$$\tfrac{9}{2}(\delta^2-2\delta^3+\delta^4)g^4\lambda^2[\Delta(0)]^2 I_1\ . \tag{3.3g}$$

Summing (3.1a)–(3.1c) with (3.3a)–(3.3g) yields

$$-E=-\tfrac{1}{2}\delta^2 g^2[\Delta(0)]^2+\tfrac{1}{2}\delta^2 g^4\lambda^2[\Delta(0)]^2 I_1\neq 0 \tag{3.4}$$

signifying that spontaneous supersymmetry breaking occurs in second order in the theory in (1.7), but not in first order.

B. Calculation of the fermion-boson mass ratio R

To order δ, we evaluate the boson and fermion self-energies for the theory in (1.7) from the graphs in Figs. 11 and 12, respectively. The diagrams in Fig. 11 yield the result that to first order the boson mass squared is

$$m_b^2=g^2\lambda^2-2\delta g^2\lambda^2\ . \tag{3.5}$$

Figure 12 gives the fermion mass as

$$m_f=g\lambda-\delta g\lambda\ . \tag{3.6}$$

From (3.5) and (3.6), we see that to order δ, the ratio

$$R=\frac{m_f}{m_b}=1 \tag{3.7}$$

which confirms our earlier result of unbroken supersymmetry to order δ for the theory in (1.7).

FIG. 12. Fermion self-energy graph of the theory in (1.7), to order δ.

FIG. 13. Two-vertex graphs of order δ^2, contributing to the boson self-energy of the theory in (1.7).

To evaluate the boson two-point function $\Pi(p)$ to order δ^2, we add the diagrams in Fig. 13 to those in Fig. 11. (All other two-vertex graphs with two external boson lines are at least of order δ^3.) The contributions of the graphs in Fig. 13 are

$$4\delta^2 g^2\Delta(0)-(8\delta^2 g^4\lambda^2+2\delta^2 g^2 p^2)I_{2,p} \ , \tag{3.8a}$$

$$18(\delta^2-2\delta^3+\delta^4)g^4\lambda^2 I_{2,p} \ . \tag{3.8b}$$

Following the same procedure as in Sec. II B, we find that to order δ^2, the mass squared of the boson of the theory in (1.7) is given by the zero of

$$p^2+g^2\lambda^2-(2\delta-\delta^2)g^2\lambda^2+2\delta^2 g^2\Delta(0)-12\delta^2 g^4\lambda^2 I_{2,p} \ . \tag{3.9}$$

We now evaluate the fermion self-energy $\Sigma(p)$ to order δ^2 by adding the contribution of Fig. 14 to that of Fig. 12. The contribution of Fig. 14 is

$$-2\delta^2 g^2 \not{p} I_{2,p}-4\delta^2 g^3\lambda I_{2,p} \ . \tag{3.10}$$

Then we find that the mass squared of the fermion is

FIG. 14. The order-δ^2 contribution to the fermion self-energy of the theory in (1.7).

given by the zero of

$$p^2+g^2\lambda^2-(2\delta-\delta^2)g^2\lambda^2-12\delta^2 g^4\lambda^2 I_{2,p} \ . \tag{3.11}$$

Comparing (3.9) and (3.11) we see that the ratio

$$R=\frac{m_f}{m_b}\neq 1 \tag{3.12}$$

to order δ^2, which confirms the result of broken supersymmetry at second order for the theory in (1.7).

We may conclude by saying that a linear interpolating action and a perturbative expansion in an artificial parameter δ allows a simple analysis of possible spontaneous supersymmetry breaking in models of the type shown in (1.2).

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