CAT 2021

Problem Sheet 4

(1) Let $L \subset K$ be a two-step filtration of a simplicial complex K. Describe how to extract the dimension of the relative homology group $H_k(K, L)$ for each $k \geq 0$ given the barcodes (with multiplicity) of this filtration.

(2) Let $a < a' < b < b'$ be four positive real numbers. What is the interleaving distance between the two $\mathbb{R}_+$-indexed interval modules $(I_\bullet^{a,b}, c_\bullet^{a,b})$ and $(I_\bullet^{a',b'}, c_\bullet^{a',b'})$?

(3) Show that every $\mathbb{R}_+$-indexed interval module is tame.

(4) Given two monotone functions $f, f' : K \to \mathbb{R}$ on a simplicial complex K, assume there exists some $\epsilon > 0$ so that $|f(\sigma) - f'(\sigma)| < \epsilon$ holds for every simplex σ of K. Letting $F_\bullet$ and $F'_\bullet$ denote the sublevelset filtrations of K with respect to f and f' respectively, show that the barcodes of $H_k(F_\bullet K)$ and $H_k(F'_\bullet K)$ have bottleneck distance at most ϵ for every $k \geq 0$.

(5) Let L be a simplicial complex and τ a simplex in L of dimension $k \geq 0$. What are the cohomology groups of L with coefficients in the skyscraper sheaf $\underline{\mathrm{Sk}}_\tau$?

(6) Find a sheaf S on a contractible simplicial complex L for which $H^1(L; S)$ is nonzero.