

MIDTERM EXAM 1

MATH 104, SECTION 007

Name:

The use of calculators, computers and similar devices is neither necessary nor permitted during this exam. Correct answers without proper justification will *not* receive full credit. Clearly highlight your answers and the steps taken to arrive at them: illegible work will not be graded. You may find the following information useful: for $|x| < 1$ and about the point 0,

$$\arctan(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1},$$

$$(1+x)^\alpha = \sum_{k=0}^{\infty} \binom{\alpha}{k} x^k, \text{ where } \binom{\alpha}{k} = \frac{\alpha(\alpha-1)\cdots(\alpha-k+1)}{k!}$$

Problem	Points	Earned
1	10	
2	10	
3	15	
4	20	
5	15	
6	20	
7	10	
8	5	
Total	100	

PROBLEM 1 (10 POINTS)

Consider the function $f(x) = x^{\ln(x)}$.

Part A. [7 points] Compute the derivative df/dx . Make sure you explain which differentiation rules you are using!

Part B. [3 points] Use a convenient linear approximation to estimate $f(1.01e)$.

PROBLEM 2 (10 POINTS)

If x and y are related by the equation $\cos(xy) = y^3 - x^2$, find $\frac{dy}{dx}$ in terms of x and y . Make sure you explain the differentiation rules which you have used!

PROBLEM 3 (15 POINTS)

Evaluate

$$\lim_{x \rightarrow 0} \frac{\sin(2x) - 2x}{x(\cos(x) - 1)}.$$

PROBLEM 4 (20 POINTS)

Consider the polynomial $f(x) = \frac{1}{2}x^4 + \frac{2}{3}x^3 - 6x^2 + 5$.

Part A. (7 Points) Find all the critical points of f .

Part B. (7 Points) Classify each critical point from **Part A** as max, min or fail.

Part C. (6 Points) Find the global max and min of f on $[-1, 1]$.

PROBLEM 5 (15 POINTS)

Consider the function $g(x) = (x - 4)^{-1/2}$.

Part A. (4 Points) What is the domain of g ?

Part B. (7 Points) What is the coefficient of the $(x - 8)^3$ term in the Taylor series of g about $x = 8$? There is no need to simplify the answer too much.

Part C. (4 Points) In which interval does the Taylor series from **Part B** converge?

PROBLEM 6 (15 POINTS)

Find the Taylor series of $f(x) = (1 + \arctan(x))^{-1/2}$ near $x = 0$, including all terms of order 3 and below.

PROBLEM 7 (10 POINTS)

Consider the functions $f(x) = \ln(x)$ and $g(x) = x^3 - 8$.

Part A. (2 Points) Find a function $h(x)$ so that $h(x) = 0$ only at those x values where $f(x) = g(x)$.

Part B. (6 Points) What is the update rule to obtain x_{n+1} from x_n when solving $h(x) = 0$ by Newton's method?

Part C. (2 Points) Use your update rule from **Part B** to compute x_1 when $x_0 = 1$.

PROBLEM 8 (5 POINTS)

This problem asks for two definitions. No partial credit will be awarded for incorrect answers.

Part A. (3 Points) Complete this sentence, using suitable ϵ 's and δ 's as necessary: *the limit $\lim_{x \rightarrow a} f(x)$ equals L if ...*

Part B. (2 Points) Fill up the box with a suitable expression for $f'(x)$:

$$f'(x) = \lim_{h \rightarrow 0} \boxed{}$$

FOR SCRATCHWORK
