

MIDTERM EXAM 2

MATH 104, SECTION 007

Name:

The use of calculators, computers and similar devices is neither necessary nor permitted during this exam. **Correct answers without proper justification will not receive full credit.** Clearly highlight your answers and the steps taken to arrive at them: illegible work will not be graded. You may find the following information useful:

$$\sin(2x) = 2 \sin(x) \cos(x) \quad \text{and} \quad \cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\int \sec x dx = \ln |\sec x + \tan x| + C \quad \text{and} \quad \int \csc x dx = -\ln |\csc x + \cot x| + C$$

Problem	Points	Earned
1	15	
2	10	
3	20	
4	15	
5	10	
6	10	
7	10	
8	10	
Total	100	

PROBLEM 1 (15 POINTS)

Consider the ordinary differential equation $\frac{dy}{dx} = y \tan(x) + \frac{\sec(x)}{x^3}$.

Part A. [8 points] What is the integrating factor for this equation?

Part B. [7 points] Use your integrating factor to find the general solution of the given equation.

PROBLEM 2 (10 POINTS)

Use a suitable technique to evaluate the definite integral

$$\int_4^5 \frac{3x + 1}{x^2 + 3x + 2} dx$$

PROBLEM 3 (20 POINTS)

Explain *carefully* whether the following improper integral converges or diverges.

$$\int_2^{\infty} \frac{dx}{(x^2 - 4)^{3/4}}$$

Point out all the Taylor series expansions (if any) which you have used to simplify your calculations as well as the p-values involved when invoking the p-test.

PROBLEM 4 (15 POINTS)

Consider the differential equation

$$\frac{dx}{dt} = (x^2 - 9)(e^{x-1} - 1).$$

Part A. (3 Points) Find all the equilibria.

Part B. (6 Points) Classify each equilibrium as stable or unstable, carefully explaining how you obtained that answer.

Part C. (3 Points) What is $\lim_{t \rightarrow +\infty} x(t)$ if $x(0) = -2$? How did you get this answer?

Part D. (3 Points) What is $\lim_{t \rightarrow -\infty} x(t)$ if $x(0) = 2$? How did you get this answer?

PROBLEM 5 (10 POINTS)

Consider the function $f(x) = \int_x^{x^2} \cos(e^{-t}) dt$.

Part A. (3 Points) Carefully state any form the **Fundamental theorem** of integral calculus. There is no partial credit here, so be careful!

Part B. (7 Points) Compute $\frac{df}{dx}$ for the function $f(x)$ mentioned above. **Hint:** please don't try to actually compute that hideous integral.

PROBLEM 6 (10 POINTS)

Evaluate the following indefinite integral using a suitable technique:

$$\int \frac{dx}{x^2 \sqrt{x^2 - 9}}.$$

PROBLEM 7 (10 POINTS)

If the quantity of money in a savings account accrues 10 percent interest every year, how many years will it take to triple the original amount?

PROBLEM 8 (10 POINTS)

Evaluate the integral

$$\int \sec^3 x \tan^5 x dx$$

FOR SCRATCHWORK
