

MIDTERM EXAM 3

MATH 104, SECTION 007

Name:

The use of calculators, computers and similar devices is neither necessary nor permitted during this exam. **Correct answers without proper justification will *not* receive full credit.** Clearly highlight your answers and the steps taken to arrive at them: illegible work will not be graded.

Problem	Points	Earned
1	15	
2	15	
3	10	
4	25	
5	10	
6	10	
7	15	
Total	100	

PROBLEM 1 (15 POINTS)

Consider the function $f(x) = \frac{\ln(x)}{x}$ defined for all $x > 0$.

Part A. [3 points] For which values $b \geq 1$ is $f(x)$ a probability density function (PDF) on $[1, b]$?

Part B. [6 points] Find the expectation $\mathbb{E}[x^2]$ when x is chosen randomly according to the PDF above.

Part C. [6 points] Set up, but *do not solve*, an expression which computes the variance $\mathbb{V}[x^2]$.

PROBLEM 2 (15 POINTS)

Let R be the region defined in the plane by $x \geq 0$, $y \geq 0$ and $y \leq 1 - \frac{x^2}{4}$

Part A. [4 Points] Find the area of R .

Part B. [5 Points] Find \bar{x} , the x -coordinate of the centroid of R .

Part C. [6 Points] Find \bar{y} , the y -coordinate of the centroid of R .

PROBLEM 3 (10 POINTS)

What is the probability that a point (x, y) sampled uniformly from the unit disk (defined by $x^2 + y^2 = 1$) satisfies $x + y > 1$?

(continue)

PROBLEM 4 (25 POINTS)

Consider the function $f(x) = \sqrt{x}$ for x between 0 and 4, and let S be the solid defined by rotating the graph of this function about the y axis.

Part A. (5 Points) What is the volume of S ?

Part B. (10 Points) Given a density $\rho(y) = \frac{1}{y^3}$, find the work done to dig a S-shaped ditch.

Part C. (10 Points) Assuming a constant density ρ , find the moment of inertia when rotating S about its central axis.

PROBLEM 5 (10 POINTS)

Consider the function $y = \frac{x^2}{4} - \frac{\ln(x)}{2}$ for $1 \leq x \leq e$.

Part A. [5 Points] Find the arclength of the graph of y .

Part B. [5 Points] Set up, but *do not solve*, an integral which computes the surface area of the graph of y rotated about the x -axis

PROBLEM 6 (10 POINTS)

Use polar coordinates to compute the area lying inside the disk of radius 2 with center $(0,0)$ for which $y \geq 1$.

PROBLEM 7 (15 POINTS)

For which interest rate $r > 0$ will the income stream $I(t) = t$ (where $0 \leq t < \infty$) have present value = 10,000?

FOR SCRATCHWORK
