DAY 1
Wed Aug 26

FUNCTIONS & EXPONENTIALS e^x

But first, ...

0. Website, CANVAS
1. Textbook (no need!)
2. Recitation (none tomorrow!)
3. Permits (DRL 4C1, office!)
4. Office Hours (TBA)
5. VIDEOS! (Coursera Upenn Calc, Prof Robert Ghrist)

README!

ALSO

Can you COMPUTE
- simple limits?
- derivatives?
- integrals?

if not, try math 103

WELCOME TO

SECTION 007 of Math 104

(For Your Engineers only?)

And here we ... go:

\# 1. What is the INVERSE of $f(x) = \sin(3x^2)$?

Ans $f(x) = $ sin of 3 times the square of x

FLIP!

the sq. root of 1/3 times arcsin of x

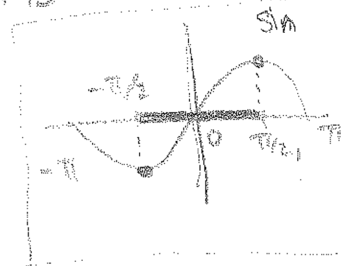
$$\boxed{f^{-1}(x) = \sqrt{\tfrac{1}{3}\arcsin(x)}}$$

That was easy! We only need to
- decompose f into a composite of easy functions:
  $f = f_1 \circ f_2 \circ f_3$
- invert and flip: $f^{-1} = f_3^{-1} \circ f_2^{-1} \circ f_1^{-1}$

#2 | Okay, smartass. What is the **DOMAIN** of your f^{-1} from before?

Ans | Well, $f^{-1}(x) = \sqrt{1/3 \arcsin x}$. Again, this **DE-COMPOSES** into:

$$x \longrightarrow \boxed{\text{ARCSIN}(\cdot)} \xrightarrow{} \underset{\arcsin x}{} \boxed{1/3\,\cdot} \xrightarrow{} \underset{1/3 \arcsin x}{} \boxed{\sqrt{\cdot}} \longrightarrow \underset{\sqrt{1/3 \arcsin x}}{f^{-1}(x)}$$



- Domain of arcsin : $[-1, 1]$
- Domain of $1/3(\cdot)$: everything
- Domain of $\sqrt{\cdot}$: $[0, \infty)$.

So: x must at least lie in $[-1, 1]$, but ALSO need $1/3 \arcsin(x) \geq 0$, so:

$$\boxed{x \text{ must lie in } [0, 1]}$$

Ha Ha!!

YOUR TURN:

Try to repeat #1 and #2, but with $\boxed{f(x) = \ln(\arccos x)}$

Moving on, ...

WHAT (THE F...) IS e^x ??

First, know thine POLYNOMIALS.

$$p(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_d x^d = \sum_{n=0}^{d} a_n x^n$$

d = degree of p.

Eg

A. $p(x) = 1$ ← this totally counts!

B. $p(x) = 3 - x^2$

C. $p(x) = 1 + x - x^2 + 4x^3 - 19x^{21}$

COMPUTING derivatives, $\frac{d}{dx} p(x)$ lowers degrees.

A. 0

B. $-2x$

C. $1 - 2x + 12x^2 - (19 \cdot 21)x^{20}$ Ugh ...

e^x is WHAT YOU GET if you try to create a non-zero polynomial whose derivative equals itself:

$$e^x = 1 + x + x^2/2! + x^3/3! + \dots = \boxed{\sum_{k=0}^{\infty} x^k/k!}$$

for ever!

("Taylor" series)

Let's CONFIRM a few things... (we already know)

1. $d/dx = e^x$.

2. $e^{x+y} = e^x \cdot e^y$.

3. $(e^x)^y = e^{xy}$.

(what is) $e^{2x} \cdot e^{3x}$?

The INVERSE of e^x is $\ln x$, **so**:

Many other identities can be derived using this...

$$\sum_{k=0}^{\infty} (\ln x)^k / k! = e^{\ln x} = x$$

(???!!)

(need $x > 0$ to define $\ln x$)

EULER'S FORMULA:

Very important...

Remember: i is a COMPLEX NUMBER, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$, etc...

$$\boxed{e^{ix} = \cos x + i \cdot \sin x}$$

From which:

Taylor series...

- $\cos x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$
- $\sin x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$

And $e^{i\pi} = -1$ $\quad e^{i\pi/2} = i$

WOW: $i^i = (e^{i\pi/2})^i = e^{i^2 \pi/2} = e^{-\pi/2} = \frac{1}{\sqrt{e^{\pi}}}$