Examples of product rule, quotient rule, and chain rule

6.

Let $f(x) = \underbrace{2x}_{g}\cdot\underbrace{e^{\sqrt{x}}}_{h}$. Find $f'(9)$.

A) $6e^3$
B) $12e^3$
C) $4e^3$
D) $10e^3$
E) $9e^3$
F) $18e^3$
G) $\frac{e^3}{2}$
H) $5e^3$

$$f'(x) = \underbrace{2}_{g'}\cdot\underbrace{e^{\sqrt{x}}}_{h} + \underbrace{\cancel{2}x}_{g}\cdot\underbrace{e^{\sqrt{x}}\cdot\frac{1}{\cancel{2}\sqrt{x}}}_{h'}$$

$$f'(9) = 2\cdot e^{\sqrt{9}} + \cancel{9}^{3}\, e^{\sqrt{9}}\cdot\frac{1}{\cancel{\sqrt{9}}_{3}}$$

$$= 2e^3 + 3e^3 = \boxed{5e^3}$$

3. Let

$$f(x) = \ln\left(\frac{x}{\sqrt{x^2+1}}\right)$$

Find $f'(2)$.

A) 1/10
B) 1/8
C) 1/40
D) 1/5
E) 1/30
F) 1/20
G) 1/50
H) 1/8

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$\ln(a^p) = p\ln a$

$\ln(a+b)$ no rule

$(x^2+1)^{1/2}$

$$f(x) = \ln x - \ln\sqrt{x^2+1}$$

$$f(x) = \ln x - \frac{1}{2}\left(\ln(x^2+1)\right)$$

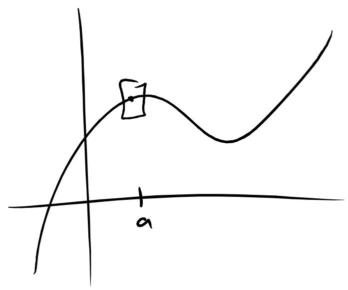
$h = \ln g(x)$

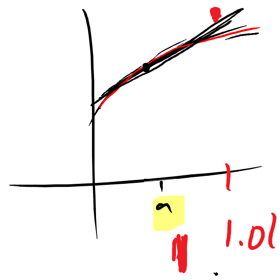
$h' = \frac{g'(x)}{g(x)}$

$$f'(x) = \frac{1}{x} - \frac{1}{\cancel{2}}\cdot\frac{1}{x^2+1}\cdot(\cancel{2}x)$$

$$f'(2) = \frac{1\cdot 5}{2\cdot 5} - \frac{2\cdot 2}{5\cdot 2} = 1/10$$

Linearization First variation





using the tangent line approximation

$$f(x+h) = \underbrace{\boxed{f(x) + \frac{df}{dx}\cdot h}}_{\text{linear}} + O(h^2)$$

Approximate $\log(1.01)$ using the first variation (linearization)

$$\log(x) = \log_{10}(x)$$

$$f(x) = \log(x)$$

$$\log(1) = 0$$

$x = 1$

$h = \frac{1}{100}$

$$f\left(1+\tfrac{1}{100}\right) = f(1) + \underbrace{\boxed{\frac{1}{1\cdot \ln 10}}}_{\frac{df}{dx}} \cdot \underbrace{\frac{1}{100}}_{h} + O(h^2)$$

$f = 2^x$

$f' = 2^x \cdot \ln 2$

$g = \log_2 x$

$g' = \dfrac{1}{x \cdot \ln 2}$

$h = \log_{10}(x)$

$h' = \dfrac{1}{x \cdot \ln 10}$

$$f\left(1+\tfrac{1}{100}\right) = f(1) + \overset{dx}{\boxed{\frac{1}{1\cdot\ln 10}}} \cdot \overset{h}{\frac{1}{100}} + O(h^2)$$

drop

$$\log(1.01) \approx 0 + \frac{1}{100\ln 10}$$

```
1/(100*ln(10))
     .0043429448
```

```
     .0043429448
log(1.01)
     .0043213738
1/(100*ln(10))
     .0043429448
log(1.01)-Ans
  -2.15710364E-5
```

} error

-0.0000215710364

Newton's Method (Application of Linearization)

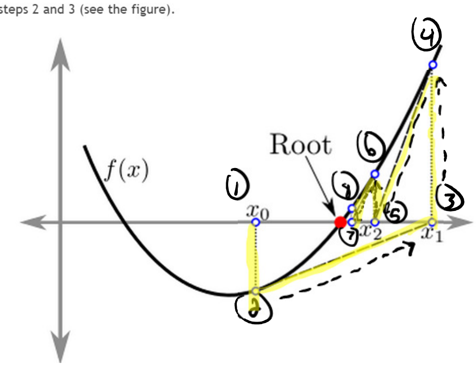
More formally, this is what is called a *difference equation*. Given an initial guess, called x_0, of a root of the function, one uses the update rule

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

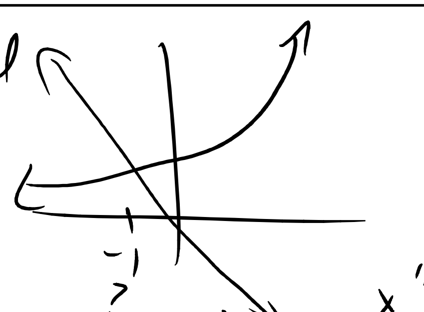
to get x_1, and then x_2, and so on.

The resulting sequence hopefully converges to a root of f. Graphically, what is happening is as follows:

1. Pick a guess x_0.
2. Find the tangent line to f through the point $(x_0, f(x_0))$.
3. Let x_1 be the point where the tangent line intersects the x-axis.
4. Repeat steps 2 and 3 (see the figure).



Newton's Method



$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

"Where does $y = -x$ meet $y = e^x$?"

$$-x = e^x$$

initial guess $x_0 = -1$

$$\boxed{f(x) = e^x + x}$$

Find root

$x_0 = -1$

$f(x) = e^x + x$

$f(-1) = e^{-1} + -1$

$f'(x) = e^x + 1$

$f'(-1) = e^{-1} + 1$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

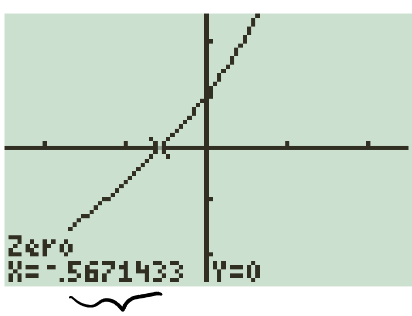
$$x_1 = -1 - \frac{f(-1)}{f'(-1)}$$

$$x_1 = -1 - \boxed{\frac{\frac{1}{e} - 1}{\frac{1}{e} + 1}} \approx -0.5378828$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

```
     -.4621171573
-1-Ans
     -.5378828427
Ans→A
     -.5378828427
A-Y1(A)/Y2(A)
     -.5669869914
```

$x_0 = -1$ $\quad x_1$ $\quad x_2$



$x_2 = -0.5669869914$

computer generated root $= -0.5671433$