

QUIZ 1

INSTRUCTIONS

Please answer the following questions to the best of your ability and understanding, without seeking help from books, notes, colleagues, the internet, etc. The use of calculators is neither permitted nor necessary. However, you might find the following information useful: for $|x| < 1$, we have:

$$\arctan(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)} \quad \text{and} \quad \frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$$

PROBLEM 1

(15 Points) What is the Taylor series of $f(x) = \frac{1}{1-\arctan(2x)}$ including all terms with order ≤ 3 ?

PROBLEM 2

(5 Points) For which values of x does the Taylor series for $f(x) = \frac{1}{x}$ about $x = 1$ converge? You *don't* need to compute terms, just figure out the interval!

PROBLEM 3

(10 Points) What is $\lim_{x \rightarrow a} \frac{x-a}{\ln x - \ln a}$ for a constant $a > 0$?

PROBLEM 4

(10 Points) Find the *largest* integer n so that $\sin(x) - \arctan(x)$ is $O(x^n)$ as $x \rightarrow 0$.

PROBLEM 5

(10 Points) What is the Taylor series (include all terms of order 2 and below) of $f(x) = x^{1/2}$ near $x = 4$? What is your best guess for the value of $\sqrt{4.4}$ obtained by using only the linear part (i.e., terms of order 0 and 1 only) of this series?