(PRACTICE) QUIZ 2

Instructions

Please answer the following questions to the best of your ability and understanding **within 30 minutes**. Do not use books, notes, the internet, calculators, etc. You might find the following information useful:

$$\cos(2x) = \cos^2(x) - \sin^2(x) \quad \text{and} \quad \int \cot(x)\,dx = \ln(\sin(x)) + C$$

Problem 1

(**10** Points) Evaluate the definite integral $\int_0^{\pi/2} x^2 \sin(2x)\,dx$ using a suitable integration technique.

Problem 2

(**10** Points) If $\frac{dx}{dt} = 3x$, at which value of t will x equal 4 times its initial value?

Problem 3

(**10** Points) Does the improper integral $\int_{-1}^{\infty} \frac{dx}{\sqrt{x^3+2}}$ converge or diverge? Carefully explain why.

Problem 4

(**10** Points) Consider the linear ODE $\frac{dy}{dx} = y\cot(x) + \sin^3(x)$.

Part A. Find the integrating factor.

Part B. Find the general solution to this ODE.

Problem 5

(**10** Points) Consider the ODE $\frac{dy}{dx} = (e^x - 1)(x^2 - 2)$.

Part A. Find all the equilibria.

Part B. Classify each equilibrium as stable or unstable.

Part C. What is $\lim_{t\to\infty} x(t)$ if $x(0) = -1$?

Part D. What is $\lim_{t\to-\infty} x(t)$ if $x(0) = 7$?