

# QUIZ 4

## INSTRUCTIONS

Please answer the following questions to the best of your ability and understanding **within 30 minutes**. Do not use books, notes, the internet, calculators, etc.

### PROBLEM 1

(10 Points) Consider the sequence  $a_n = \left(\frac{n}{n+2}\right)^n$ .

**Part A.** (6 Points) Either compute  $\lim_{n \rightarrow \infty} a_n$ , or explain why this sequence diverges.

**Part B.** (4 Points) Does the series  $\sum_{n=0}^{\infty} a_n$  converge or diverge? Explain why.

## PROBLEM 2

(15 Points) Carefully explain whether the following series converge or diverge, making sure that you mention which convergence test(s) have been used.

**Part A.** (5 Points)  $\sum_{n=1}^{\infty} n^2 \left( e^{-1/n^3} - 1 \right)$

**Part B.** (5 Points)  $\sum_{n=1}^{\infty} (-1)^n \left[ \left( \frac{n}{n+2} \right)^n - 1 \right]$  (**Hint:** it will help if you have solved Problem 1 first).

**Part C.** (5 Points)  $\sum_{n=1}^{\infty} \frac{2^n \ln(n)}{(2n)!}$

## PROBLEM 3

(15 Points) Consider the power series  $f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{1-3n^2}$ .

**Part A.** (8 Points) Find the interval of convergence.

**Part B.** (7 Points) Use any convenient method to find a suitable  $N$  so that the error when approximating  $f(x)$  by the first  $N$  terms of its power series is guaranteed to be smaller than 0.01.

## PROBLEM 4

(10 Points) Five series are given below. Write down which of them converge absolutely, converge conditionally, or diverge. You don't have to show much work here, just a brief line (eg: diverges by limit comparison to  $\sum \frac{1}{n}$ , or diverges by ratio test) will suffice. Each answer is worth two points, but there is **no partial credit** for incorrect responses.

**Part A.**  $\sum_{n=1}^{\infty} \frac{n - \ln(n)}{\sqrt[3]{n^2 + n - 7 \ln(n+5)}}$

**Part B.**  $\sum_{n=1}^{\infty} \left( \frac{n^2 - 1}{n^2 + 3} \right)^n$

**Part C.**  $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n+1}}{\sqrt[3]{n^2 - 5}}$

**Part D.**  $\sum_{n=1}^{\infty} \frac{3^n}{5^n - n^3}$

**Part E.**  $\sum_{n=1}^{\infty} \frac{\cos^3(e^n - 28n^2)}{n^2 + 2n}$