

### PROBLEM 1

**Part a.** One point for knowing  $A = LU$ , another two for correct multiplication.

**Part b.** One point for each correct row operation.

**Part c.** Five points for setting up the correct triangular systems and five for the correct order.

**Part d.** Three points for setting up  $[A \mid \text{Id}]$  or  $[\text{Id} \mid A]$ , then another two for the correct inverse

**Part e.** Two points for recognizing  $A^{-1} = U^{-1}L^{-1}$  and another two for the correct multiplication. **Or**, two points for setting up the Gauss-Jordan elimination and another two for the correct solution.

### PROBLEM 2

Throughout this problem, just producing a correct basis with no explanation whatsoever resulted in the loss of roughly half the points. If the unexplained basis also happened to be incorrect, then no points were awarded.

**Part a.** One point for recognizing that the row space is spanned by pivot rows of  $R$ , and four for correctly extracting these two rows.

**Part b.** Five points for recognizing that the column space of  $B$  is just the column space of  $R$  mapped forward by  $M$  and another five for correctly extracting the columns of  $M$  corresponding to the pivots of  $R$ .

**Part c.** Five points for correctly setting up the free and pivot variables, another five for a correct expression for vectors in the nullspace.

**Part d.** Five points for observing that a basis for the left null space is given by the rows of  $M^{-1}$  corresponding to the zero rows of  $R$ . Another five for correctly identifying the third row of  $M^{-1}$  as a basis vector.

**Part e.** One point for stating the fundamental theorem correctly (this point was lost if you used  $m, n$  and  $r$  without explaining what those are). One point each for correctly extracting the dimensions of the four fundamental subspaces of  $B$ .

### PROBLEM 3

**Part a.** Two points for computing  $M^{-1}\mathbf{v}$ , another two for the correct reduced row echelon form of the augmented matrix, and three for recognizing that there were no solutions.

**Part b.** Two points for computing  $M^{-1}\mathbf{v}$ , another two for the correct reduced row echelon form of the augmented matrix, and one for getting a particular solution, and another two for the final answer which consists of particular solution plus null space.

### PROBLEM 4

For each part, one point for correctly identifying the given statement as true or false, and another two for a valid justification of your answer.