Problem 1

Part a. One point per correct entry of A.

Part b. One point for the correct polynomial, one for each eigenvalue, and three for each eigenvector.

Part c. Two points for $e^{At} = Se^{Dt}S^{-1}$, one for the correct S^{-1}, and one for correct matrix multiplication.

Part d. Five points for knowing $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = e^{At} \begin{bmatrix} x(0) \\ y(0) \end{bmatrix}$, one for the correct $\begin{bmatrix} x(0) \\ y(0) \end{bmatrix}$, and two for correct multiplication. Severe penalties for those who made $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ equal a 2×2 matrix rather than a vector. What on earth does that even mean?

Problem 2

Part a. One point for each eigenvalue, one for each eigenvector, two for normalizing correctly.

Part b. Two points for knowing that the nonzero eigenvalues of AA^T coincide with those of A^TA from **Part a**, and another one point for recognizing that the third eigenvalue is 0. If you did the entire computation starting from $\det(AA^T - \lambda I) = 0$ then your score depends on how far you got: if you really did get the three correct eigenvalues this way, then you got all three points.

Part c. Two points per correct eigenvector, computed either via inspection or via row operations. Two points were allocated for normalizing all three eigenvectors correctly.

Part d. Five points each for correct D, U and V: in each case, three points for knowing what entries to put in each matrix (eg, V contains the eigenvectors of ...), and two points for actually putting these entries in the right place in the right order.

Part e. Two points per correct basis: one for knowing which part of the SVD to get it from, and another for actually getting the correct basis from that part of the SVD. Severe penalties were handed to those who didn't even get the dimensions of these four subspaces right.

Problem 3

Part a. One points for computing the first vector (including correct normalization), two points for the correct projection of the second onto the first, and one point for the correct second vector – normalized.

Part b. Just knowing that Q's columns contain the two vectors obtained in **Part a** would get you all five points. However, no points were awarded to those whose Q was not even orthonormal.

Part c. Knowing that the least squares solution is given by $Q^T \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ would earn five points, and actually carrying out the computation would get another five. If you started with the normal equation $Q^TQ \left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] = Q^T \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, you still got three points. In this case, the remaining seven were awarded if you actually computed $Q^TQ = I$, then $Q^T \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, and hence the correct answer.

Problem 4

In each case, one point for the correct answer (true or false) and two for a correct justification. For **Part c**, just saying "row operations change eigenvalues" without providing a valid counter-example resulted in the loss of one point: just because row operations change eigenvalues in general doesn't mean that the explicit row operation mentioned in the question changes eigenvalues.