HOMEWORK ASSIGNMENT 2

Name: **Due:** Monday Feb 10

Problem 1: Strang 2.3 #3 page 63

Which three matrices E_{21}, E_{31} and E_{32} put A into (upper) triangular form U? Here,

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix}$$

and we want $E_{32}E_{31}E_{21}A = U$. Multiply these three E matrices to get a single matrix M that does the elimination: $MA = U$.

Ans:

Problem 2: Strang 2.3 #10 page 64

Answer the three questions below:

(a) What 3 by 3 matrix will add row 3 to row 1?
(b) What matrix adds row 1 to row 3 and *at the same time* row 3 to row 1?
(c) What matrix adds row 1 to row 3 and *then* adds row 3 to row 1?

(Additional Question): Which of the matrices from (a), (b) and (c) are *not* invertible? Explain.

Ans:

Problem 3: Strang 2.4 #5 page 76

Compute A^2 and A^3 in each of the following two cases, and then make predictions for A^5 and A^n:

$$A = \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} \text{ and } A = \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix}.$$

Ans:

Problem 4: Strang 2.4 #32 page 80

Suppose you solve $Ax = b$ for three special right sides b:

$$Ax_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ and } Ax_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \text{ and } Ax_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

If the three solutions x_1, x_2 and x_3 are the columns of a matrix X, what is the matrix product AX?

Ans:

Problem 5: Strang 2.5 #25 page 91

Find A^{-1} and B^{-1} (*if they exist!*) by using Gauss-Jordan elimination. Otherwise, explain why the matrix is not invertible.

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}.$$

Ans:

Problem 6: Strang 2.5 #9 page 89

Suppose that the matrix A is invertible and you exchange its first two rows to get B. Is the new matrix B also invertible? If yes, explain how you would find B^{-1} from A^{-1} and if no, give an example that shows B need not be invertible.
Ans:

Problem 7: Strang 2.6 #7 page 103

What three elimination matrices E_{21}, E_{31} and E_{32} put A into upper triangular form? Multiply by E_{32}^{-1}, E_{31}^{-1} and E_{21}^{-1} to factor A into $LU = \left(E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}\right)U$.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix}.$$

Ans:

Problem 8: Strang 2.6 #16 page 105

The LU decomposition of an unknown matrix A is

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \text{ and } U = L^T.$$

Here, U is the transpose of L. First solve the matrix equation $Ax = b$ for $b = \left[\begin{smallmatrix} 4 \\ 5 \\ 6 \end{smallmatrix}\right]$ as two triangular systems. Then, compute the original matrix A.

Ans: