HOMEWORK ASSIGNMENT 3

Name: **Due:** Monday Feb 24

Problem 1: Strang 3.1 #9, #10 page 128

This problems tests your understanding of vector spaces and subspaces.

(1) Find a set of vectors in $\mathbb{R}^2$ for which $x+y$ stays in the set but $\frac{1}{2}x$ may be outside for some x in the set.
(2) Find a set of vectors in $\mathbb{R}^2$ (other than two quarter-planes) for which every cx stays in the set but $x+y$ may be outside.
(3) Is the set of vectors (b_1, b_2, b_3) with $b_1 = b_2$ a subspace of $\mathbb{R}^3$? Briefly explain why or why not.
(4) Same as above, the set of vectors with $b_1 b_2 b_3 = 0$.
(5) Same as above, the set of vectors with $b_1 \leqslant b_2 \leqslant b_3$.

Ans:

Problem 2: Strang 3.4 #1 page 163

Describe the column space and null space of A. Also compute the complete solution to $Ax = b$

$$A = \begin{bmatrix} 2 & 4 & 6 & 4 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & 2 \end{bmatrix}, b = \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}.$$

Ans:

Problem 3: Strang 3.4 #8 page 164

Which vectors $[b_1, b_2, b_3]$ are in the column space of A? Which combinations of rows of A give the zero row? Answer these questions separately for these two choices of A:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 6 & 3 \\ 0 & 2 & 5 \end{bmatrix}, \qquad A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix}.$$

Ans:

Problem 4: Strang 3.4 #18 page 165

Compute the ranks of A and A^T (these might depend on q). Show your work!

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & q \end{bmatrix}.$$

Ans:

Problem 5: Strang 3.5 #2 page 178

Find the largest possible number of linearly independent vectors among

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, v_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, v_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}.$$

Explain how you found this number.

Ans:

Problem 6: Strang 3.5 #25 page 179

Decide the dependence or independence of

(1) the vectors $(1, 3, 2)$, $(2, 1, 3)$ and $(3, 2, 1)$,
(2) the vectors $(1, -3, 2)$, $(2, 1, -3)$ and $(-3, 2, 1)$.

Again, explain your answers.

Ans:

Problem 6: Not from Strang

The vector $b = (4, 20, 14)$ equals $-3u + v + 5w$ where $u = (2, 4, 1)$, $v = (0, 2, 2)$ and $w = (2, 6, 3)$. Can we do any better? Does b lie in the span of u and v alone? Explain why or why not.

Ans:

Problem 7: Strang 3.5 #20 page 180

Find a basis for the plane $x - 2y + 3z = 0$ in $\mathbb{R}^3$. Then find a basis for the intersection of that plane with the xy plane. (Hint: both problems can be expressed in terms of finding nullspaces of certain matrices).
Ans:

Problem 8: Not in Strang

All you know about a 3×5 matrix A is that it has rank 2. Compute $\dim N(A) - \dim N(A^T) + \dim C(A) - \dim C(A^T)$. Explain how you got your answer.

Problem 9: Strang 3.6 #3 page 191

Find bases for each of the four fundamental subspaces associated with A.

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Ans: