HOMEWORK ASSIGNMENT 4

Name: **Due:** Wednesday Mar 19

Problem 1: Strang 4.1 #6 page 203

This system of equations $Ax = b$ has no solutions.

$$\begin{aligned} x + 2y + 2z &= 5 \\ 2x + 2y + 3z &= 5 \\ 3x + 4y + 5z &= 9. \end{aligned}$$

(1) Find numbers y_1, y_2 and y_3 so that scaling the first equation by y_1, the second by y_2 and the third by y_3 before adding them all up leads to the contradiction $0 = 1$.
(2) Which of A's four fundamental subspaces contains the vector $y = (y_1, y_2, y_3)$?

Ans:

Problem 2: Strang 4.1 #11 page 203

Draw and label the four fundamental subspaces for

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}.$$

Draw the row and null space in one figure and the column and left null space in another.

Ans:

Problem 3: Strang 4.1 #22 page 204

Suppose V is spanned by the vectors $(1, 2, 2, 3)$ and $(1, 3, 3, 2)$. Find a basis for $V^{\perp}$. This is the same as solving $Ax = 0$ for which matrix A?

Problem 4: Strang 4.2 #10 Page 215

Given

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \text{ and } b = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix},$$

(1) Find the matrix P which projects onto the column space of A,
(2) Compute the projection p of b onto this column space,
(3) Find the error $e = b - p$ and show that it lies in the left nullspace of A.

Ans:

Problem 5: Strang 4.2 #17 page 215

If P is a square matrix with $\mathsf{P}^2 = \mathsf{P}$, show that $(\mathrm{I} - \mathsf{P})^2 = (\mathrm{I} - \mathsf{P})$ where I is the identity matrix. Hint: just multiply out $(\mathrm{I} - \mathsf{P})(\mathrm{I} - \mathsf{P})$ and use the information given already.

Ans:

Problem 6: Strang 4.2 #19 page 216

Choose two independent vectors lying on the plane $x - y - 2z = 0$ and make them the columns of a matrix A. Then compute the matrix $\mathsf{A}(\mathsf{A}^\mathsf{T}\mathsf{A})^{-1}\mathsf{A}^\mathsf{T}$: this matrix projects onto our plane!

Ans:

Problem 7: Strang 4.3 #6 page 227

Compute the projection of $\mathbf{b} = (0, 8, 8, 20)$ onto the line through $\mathbf{a} = (1, 1, 1, 1)$ by first finding the scalar $c = \frac{\mathbf{a}^T\mathbf{b}}{\mathbf{a}^T\mathbf{a}}$.

Ans:

Problem 8: Strang 4.3 #9 page 227

Use the method of least squares to find the parabola $y = C + Dx + Ex^2$ which best approximates the four data points given in (x, y) format by $(0, 0), (1, 8), (3, 8)$ and $(4, 20)$.
Ans:

Problem 9: Strang 4.4 #6 page 240

If Q_1 and Q_2 are orthogonal matrices, show that their product $Q_1 Q_2$ is also orthogonal. Hint: use the fact that $Q^T Q$ is the identity whenever Q is orthogonal.

Ans:

Problem 10: Similar to Strang 4.4 #11 page 240

Use the Gram-Schmidt method to find orthonormal vectors q_1 and q_2 in the plane spanned by $(1, 0, -1, 1, 3)$ and $(2, 3, 2, 0, 1)$.

Ans:

Problem 11: Strang 4.4 #15 page 241

Given the matrix

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ -2 & 4 \end{bmatrix},$$

(1) Find three orthonormal vectors q_1, q_2 and q_3 so that q_1 and q_2 span the column space of A.
(2) Which of the four fundamental subspaces contains q_3?
(3) Solve $Ax = \begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}$ by least squares. Hint: it will *greatly* simplify computations if you use the orthonormal basis for $C(A)$!

Ans:

Problem 12: Strang 4.4 #31 page 243

Consider the matrix

$$Q = c \begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 \end{bmatrix}.$$

(1) Choose c so that Q becomes an orthogonal matrix.
(2) Project $b = (1, 1, 1, 1)$ onto the line spanned by the first column of Q.
(3) Project b onto the plane spanned by the first two columns of Q.