HOMEWORK ASSIGNMENT 5

Name: **Due:** Wednesday Mar 26

PROBLEM 1: STRANG 5.1 #3 AND #28 PAGE 251, 254

State true or false, giving a reason when the statement is true and a counterexample when the statement is false. All matrices involved are $n \times n$ where $n > 1$.

(1) $\det(I + A) = 1 + \det(A)$.
(2) $\det(ABC) = \det(A)\det(B)\det(C)$.
(3) $\det(4A) = 4\det(A)$.
(4) $\det(AB - BA) = 0$.
(5) If A is not invertible, AB is not invertible.
(6) $\det(A)$ always equals the product of its pivots.
(7) $\det(A - B) = \det(A) - \det(B)$.
(8) $\det(AB) = \det(BA)$.

Ans:

Problem 2: Strang 5.1 #8 page 252

Prove that every orthogonal $n \times n$ matrix Q has determinant equal to 1 or -1. **Hint:** Use the fact that $QQ^T = I$, the product formula for determinants.

Ans:

Problem 3: Strang 5.1 #24 page 254

The matrix A has the following LU factorization:

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 4 & 1 \end{bmatrix}, \text{ and } U = \begin{bmatrix} 3 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & -1 \end{bmatrix}.$$

Doing *as little computation as possible*, find the determinants of L, U, A, U^{-1}, L^{-1} and $U^{-1}L^{-1}A$.

Problem 4: Strang 5.1 #27 Page 254

Given

$$C = \begin{bmatrix} a & a & a \\ a & b & b \\ a & b & c \end{bmatrix},$$

use row operations to compute $\det(C)$.

Ans:

Problem 5: Strang 5.2 #12 Page 264

Given

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & -1 & 2 \end{bmatrix}$$

find the cofactor matrix C and compute the matrix product AC^T. Use this product to find $\det(A)$.

Ans: