HOMEWORK ASSIGNMENT 6

Name: **Due:** Monday Apr 7

Problem 1: Strang 6.2 #1 and #2 page 307

Solve the following two problems.

(1) Find matrices S and D which diagonalize $A = \left[\begin{smallmatrix} 1 & 2 \\ 0 & 3 \end{smallmatrix}\right]$ into SDS^{-1}

(2) A 2×2 matrix B has eigenvalue $\lambda_1 = 2$ with corresponding eigenvector $v_1 = \left[\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right]$ and eigenvalue $\lambda_2 = 5$ with corresponding eigenvector $v_2 = \left[\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}\right]$. What is B?

Ans:

Problem 2: Strang 6.2 #4 page 308

Let A be a matrix whose eigenvectors are all linearly independent, and let S be the matrix containing those eigenvectors as columns. State true or false for each of the following, giving reasons whenever true and counterexamples whenever false.

(a) A is invertible (b) A is diagonalizable

(c) S is invertible (d) S is diagonalizable

Ans:

Problem 3: Strang 6.2 #18 page 309

$$\text{If } A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}, \text{ show that } A^k = \frac{1}{2}\begin{bmatrix} 1+3^k & 1-3^k \\ 1-3^k & 1+3^k \end{bmatrix}.$$

Hint: Use $A^k = SD^kS^{-1}$.

Ans.

Problem 4: Strang 6.3 #1 Page 325

Solve the linear differential system $x' = Ax$ where

$$A = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix},$$

and $x(0) = \left[\begin{smallmatrix} 5 \\ -2 \end{smallmatrix}\right]$.

Ans:

Problem 5: Strang 6.3 #10 Page 326

You are given the second-order differential equation $y'' = 5y' + 4y$ with initial conditions $y(0) = 1$ and $y'(0) = 0$. Solve this equation as follows: introduce a new variable $x = y'$, so that the original equation becomes $x' = 5x + 4y$. Now solve the linear differential system

$$\begin{bmatrix} x \\ y \end{bmatrix}' = \begin{bmatrix} 5 & 4 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

with initial conditions $\begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Ans:

Problem 6: Strang 6.4 #5 Page 338

Find an orthogonal matrix Q which diagonalizes

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & -2 \\ 2 & -2 & 0 \end{bmatrix}.$$

That is, $Q^T A Q$ must be a diagonal matrix.

Ans:

Problem 7: Strang 6.4 #21 Page 340

State true (with reason) or false (with counterexample):

(1) Any matrix with real eigenvalues and eigenvectors must be symmetric.
(2) A matrix with real eigenvalues and orthogonal eigenvectors is symmetric.
(3) If a symmetric matrix is invertible, then its inverse is also symmetric.
(4) The eigenvector matrix S of a symmetric matrix is symmetric.

Ans:

Problem 7: Strang 6.7 #6 Page 372

Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

(1) Compute the singular value decomposition of A by producing the matrices U, D, V for which $A = UDV^T$.
(2) Use this SVD to read off orthonormal bases for the four fundamental subspaces associated to A.

(Continue answer to #7 here if you need extra space)