HOMEWORK ASSIGNMENT 7

Name: **Due:** Friday Apr 25

Problem 1

Only one of the following two statements is true. Identify that statement and prove it. Then provide a counter-example to the false statement.

(1) if A is a stochastic matrix, then so is A^k for any $k > 1$.
(2) if A is a stochastic matrix, then so is kA for any $k > 1$.

Ans:

Problem 2: Strang 8.3 #3 page 437

Find the eigenvalues of the following stochastic matrix:

$$A = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/4 & 1/2 & 1/4 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}.$$

What is the steady-state of the Markov chain of A starting at $p_0 = \begin{bmatrix} 1/2 \\ 1/2 \\ 0 \end{bmatrix}$?

Ans:

Problem 3

If $A > 0$ is an $n \times n$ symmetric stochastic matrix, show that the steady state for all its Markov chains is $p_\infty = \begin{bmatrix} 1/n \\ \vdots \\ 1/n \end{bmatrix}$.

Problem 4

Remember that the Perron-Frobenius theorem states that if a square matrix A has strictly positive entries, then we may derive the following four consequences:

(1) A has a non-repeated eigenvalue λ_1 which equals the spectral radius $\rho(A) > 0$,
(2) all other eigenvalues λ of A satisfy the strict inequality $|\lambda| < \lambda_1$,
(3) an eigenvector v_1 of λ_1 may be chosen to have strictly positive entries, and
(4) no other eigenvector of A can be chosen to have strictly positive entries.

Verify that the PF theorem holds for the matrix $A = \left[\begin{smallmatrix} 3 & 1 \\ 2 & 4 \end{smallmatrix}\right]$. In particular, identify λ_1 and $v_1 > 0$, then show that $|\lambda_2| < \lambda_1$ and that no scalar multiple of v_2 is strictly positive.

Ans:

Problem 5

The following matrix gives partial information about weather in Pennsylvania. There are only three types: rain, cloud and snow.

$$A = \begin{array}{c} \\ r \\ c \\ s \end{array} \begin{array}{c} \begin{array}{ccc} r & c & s \end{array} \\ \begin{bmatrix} .1 & .2 & .8 \\ .3 & .6 & .1 \\ a & b & c \end{bmatrix} \end{array}.$$

The columns contain probabilities as usual: if it rains today, then the probability of rain tomorrow is 0.1, and of cloudy skies is 0.3. Find a, b and c to complete the matrix. Then, determine the long-term probabilities of having rainy, cloudy and snowy weather by computing a suitable steady-state vector for A.

Ans: