HOMEWORK ASSIGNMENT 1

Name: **Due:** Friday Jan 31

Problem 1: Strang 2.2 #4 page 52

What multiple of the first equation should be *subtracted* from the second equation in order to proceed with the Gaussian elimination algorithm?

$$2x - 4y = 6$$
$$-x + 5y = 0.$$

After this elimination step, solve the resulting triangular system. If the right side of the first equation changes to -6 (from the current 6), what is the new solution?

Ans: $-\frac{1}{2}$ multiple

$$\begin{cases} 2x - 4y = 6 \\ 0x + 3y = 3 \end{cases}$$

$$\Rightarrow \begin{cases} x = 5 \\ y = 1 \end{cases}$$

Problem 2: Strang 2.2 #5 page 52

Choose a right side for the second equation which gives no solution and another right side which gives infinitely many solutions to the following linear system. What are two of these solutions?

$$3x + 2y = 10$$
$$6x + 4y =$$

Ans:

if equation 2 is <u>not</u> a multiple of equation 1 then there is no solution so any right side other than <u>20</u>.

On the other hand, when $6x+4y=20$, it is 2 times equation 1, the system has infinite solutions.

Problem 3: Strang 2.2 #6 page 52

Choose a coefficient b which makes the following system singular (i.e., degenerate). Then choose a g on the right side which gives infinitely many solutions,

$$2x + by = 16$$
$$4x + 8y = g.$$

Ans:

$$\begin{vmatrix} 2 & b \\ 4 & 8 \end{vmatrix} = 16-4b$$

singular $\Rightarrow$ $16-4b=0 \Rightarrow b=4$.

for infinitely many solution. eq.1 and eq.2 are multiple of each other. in this case. we compare the coefficient of x. $\Rightarrow$ eq.2 is 2-times eq.1.

so $g = 2 \cdot 16 = 32$.

Problem 4: Strang 2.2 #7 page 52

For which a does elimination break down (1) temporarily and (2) permanently?

$$ax + 3y = -3$$
$$4x + 6y = 6.$$

Solve for x and y after fixing the temporary breakdown in case (1) with a suitable row exchange.

Ans:

1) $a=0$ makes it temporarily break down.

rewrite it as: $\begin{cases} 4x + 6y = 6 \\ 3y = -3 \end{cases}$

we solve it $\begin{cases} x = 3 \\ y = -1 \end{cases}$

2) when eq.2 is multiple of eq.1 (without right side) the Gauss elimination would break down

compare y-coefficient. we get

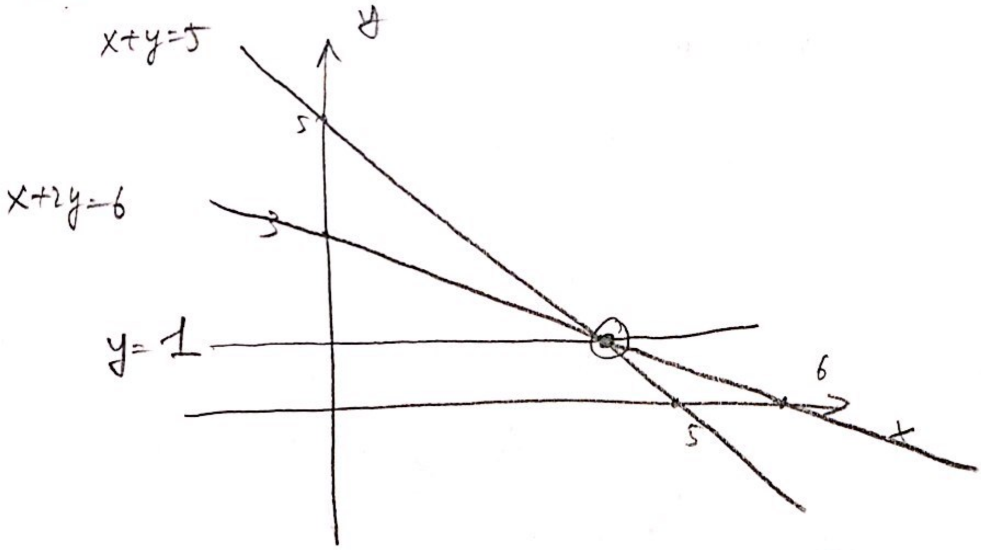
$2a = 4 \Rightarrow a = 2$

but $6 \neq 2 \cdot (-3)$. so Gauss Elimination does break down

Problem 5: Strang 2.2 #10 page 52

Draw the lines $x+y=5$ and $x+2y=6$ in the xy plane. Then draw the line $y=$ ___ which comes from elimination. The line $5x-4y=c$ will go through the solution of these equations if $c=$ ___.

Ans:



$$\begin{cases} x+y=5 \\ x+2y=6 \end{cases} \Rightarrow \begin{cases} x+y=5 \\ y=1 \end{cases} \Rightarrow \begin{cases} x=4 \\ y=1 \end{cases}$$

$$c = 5\cdot 4 - 4\cdot 1 = 16.$$

Problem 6: Strang 2.2 #13 page 53

Apply Gaussian elimination (circle the pivots) and then back-substitute to solve for x, y, z in the following linear system:

$$\begin{aligned} 2x - 3y \qquad &= 3 \\ 4x - 5y + z &= 7 \\ 2x - y - 3z &= 5. \end{aligned}$$

List the *three* operations that you used in the following format: Subtract ___ times Equation ___ from Equation ___.

Ans:

$$\left(\begin{array}{ccc|c} 2 & -3 & 0 & 3 \\ 4 & -5 & 1 & 7 \\ 2 & -1 & -3 & 5 \end{array}\right) \xrightarrow[R_3' = R_3 - R_1]{R_2' = R_2 - 2R_1} \left(\begin{array}{ccc|c} 2 & -3 & 0 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 2 & -3 & 2 \end{array}\right)$$

$$\downarrow \; R_3'' = R_3' - 2R_2'$$

$$\begin{cases} x = 3. \\ y = 1 \\ z = 0 \end{cases} \longleftarrow \left(\begin{array}{ccc|c} 2 & -3 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -5 & 0 \end{array}\right)$$

Problem 7: Strang 2.2 #19 page 54

Which number q makes the following system singular (i.e., degenerate) and which number t gives infinitely many solutions for that choice of q?

$$\begin{aligned} x + 4y - 2z &= 1 \\ x + 7y - 6z &= 6 \\ 3y + qz &= t. \end{aligned}$$

Ans:

$$\begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -6 \\ 0 & 3 & q \end{vmatrix} = \begin{vmatrix} 1 & 4 & -2 \\ 0 & 3 & -4 \\ 0 & 3 & q \end{vmatrix} = \begin{vmatrix} 1 & 4 & -2 \\ 0 & 3 & -4 \\ 0 & 0 & q+4 \end{vmatrix} = 3(q+4)$$

the system is singular $\Rightarrow$ $3(q+4)=0 \Rightarrow q=-4$.

in this case. $q=-4$. we have $R_3 = R_2 - R_1$ would make the system infinitely many solutions. so

$t = 6-1=5$.

Problem 8

Write the equations of *three planes* in 3-dimensional space so that

(1) no two of them are parallel and yet their common intersection is empty,
(2) all three of them meet in a single line.

Ans:

(1) $x=0$, $y=0$, $x+y=1$

(2) $x=0$, $y=0$, $x+y=0$.

there are lots of such constructions. the key is to make $R_3 = R_1 + R_2$ on both sides and $R_3 = R_1 + R_2$ only on the left-side but keep in mind non of the pairs can be parallel!!