Research Program

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I am interested in a broad range of scientific phenomena across many disciplines, and my goal as an applied mathematician is to use mathematical tools to understand them better by attempting to explain observed dynamics and predict novel dynamics. In my research, I use methods from subjects such as dynamical systems and perturbation theory, as most of the models I study are described by nonlinear ordinary or partial differential equations. I particularly seek problems that can be attacked both analytically and computationally, as these approaches offer complementary insights.

To find mathematical problems, I interact as much as possible with both theorists and experimentalists. When I was a VIGRE postdoctoral scholar in the Georgia Tech mathematics department, I made a point to develop strong working relationships with faculty in other departments, such as physics, biology, and electrical engineering. My current postdoctoral position in Caltech's Center for the Physics of Information, a campus initiative that includes faculty members from several departments, has allowed me to continue my interdisciplinary focus.

CURRENT RESEARCH

I use my interdisciplinary training in applied mathematics and dynamical systems to study a wide range of problems, especially in atomic, optical, and condensed matter physics.

For my doctoral research at Cornell, I investigated quantum billiards with time-dependent boundaries, which provide a mixed quantum-classical description for the dynamics of small polyatomic molecules (see, e.g., Refs. [7,16]). As a postdoctoral scholar, I have continued pursuing quantum-mechanical problems by studying Bose-Einstein condensates using a dynamical systems approach.

I have also been extremely active in advising undergraduate student research projects, often in collaboration with other faculty. Some of my students' problems, such as billiard systems with mixed (regular and chaotic) dynamics, are directly related to the interest in classical and quantum Hamiltonian systems I developed as a graduate student. For other student projects, I have branched out to new topics, such as complex networks, pattern formation, and mathematical biology.

Applications of Nonlinear Schrödinger (NLS) equations

During my postdoctoral career, my main research thrust has been the study of nonlinear Schrödinger (NLS) equations, which model the dynamics of Bose-Einstein condensates (BECs) and beam propagation in optical fibers (among other phenomena). In recognition of my work, I was invited to co-author a review article on nonlinear lattice models of BECs for *Chaos*'s 2005 focus issue on the 50th anniversary of the Fermi-Pasta-Ulam (FPU) problem [8]. I was also invited to give a talk at the International Conference on Chaos and Dynamical Complexity (May 2005), sponsored by The National Center for Theoretical Sciences in Taiwan, and in the FPU session at the 2005 American Physical Society March Meeting.

BOSE-EINSTEIN CONDENSATES (BECs): BECs are formed at extremely low temperatures when particles in a dilute gas of bosons condense into the ground state. Since they were first observed in 1995, BECs have generated considerable excitement among applied mathematicians and atomic (and condensed matter) physicists, as their extreme controllability provides an incredible amount of flexibility in both experimental and theoretical studies. (Properties such as the trapping potential, nonlinearity strength, and so on can be adjusted with a great deal of precision.) Their study has begun to yield a better understanding of superfluidity and superconductivity, and their eventual engineering applications also hold great promise. For instance, BEC research may eventually lead to more precise measurements of fundamental physical constants and improvements in the atomic clock. It is also hoped that BECs can ultimately be used in quantum computers.

The mean-field (macroscopic) dynamics of BECs are governed by the Gross-Pitaevskii (GP) equation, which consists of the cubic NLS equation plus an external potential,

$$i\frac{\partial\psi}{\partial t} = -\nabla^2\psi + g|\psi|^2\psi + V(x,t)\psi\,, \tag{1}$$

where $\psi(x,t)$ ($x \in \mathbf{R}^3$) is the condensate wavefunction, g can be either positive (for "repulsive" condensates) or negative (for "attractive" ones), ∇^2 is the Laplacian, $V(x,t)$ is the external trapping potential, and the atomic mass and Planck's constant have been scaled out. Multiple-component BECs are described by coupled GP equations.

I have concentrated my investigations on "cigar-shaped" BECs, modeled by an NLS equation with one spatial dimension, in the presence of periodic potentials $V(x,t)$, which can either be lattices [e.g., $V = V_0 \cos(\kappa x)$] or superlattices [e.g., $V = V_1 \cos(\kappa_1 x) + V_2 \cos(\kappa_2 x + x_0)$] and have been created in many laboratories. I have investigated both time-independent and time-dependent potentials and have studied Eq. (1) with constant, time-dependent, and space-dependent nonlinearity coefficients.

With P. Cvitanović, I used Hamiltonian perturbation theory to study spatial resonances of BEC coherent structures in lattice potentials [10,11]. We derived an analytical construction of "period-multiplied" wavefunctions, whose periods are integer multiples of the underlying lattice period, and anticipated subsequent experiments in which the simplest of these novel solutions (period-doubled wavefunctions) were observed for the first time (*Phys. Rev. Lett.* 95: 170404, 2005). More recently, my collaborators and I used multiple-scale perturbation theory to construct wavefunctions whose periodicity is a fraction of that of the periodic lattice [6]. With P. G. Kevrekidis and B. A. Malomed, I used averaging to construct and study the stability of resonant and non-resonant coherent structures in multiple-component BECs [15].

With P. G. Kevrekidis, I studied spatially extended wavefunctions for BECs in superlattice potentials using multiple scale perturbation theory [12]. With P. G. Kevrekidis, R. Carretero-González, and D. J. Frantzeskakis, I studied the manipulation of BEC solitary waves in superlattices, indicating techniques for controllable path-following and transfer between lattice wells (see Fig. 1). With D. E. Pelinovsky and M. Chugunova, I investigated BECs in periodic potentials under the effect of periodic temporal modulation (that is, "nonlinearity management") of the coefficient g of the cubic term in Eq. 1 [9]. With M. van Noort, Y. Yi, and S.-N. Chow, I used KAM theory to investigate quasiperiodic versus chaotic spatial dynamics in BEC coherent structures [19]. With one of my students (V. P. Chua), I complemented this approach by studying spatial resonance overlap using Chirikov's "overlap criterion" [3].

Very recently, my collaborators and I investigated BECs under spatial nonlinearity management, in which the coefficient $g = g(x)$ is a periodic function of space [13]. Among our results are that "on-site" solutions, whose maxima correspond to maxima of $g(x)$, are significantly more stable than "off-site" solutions.

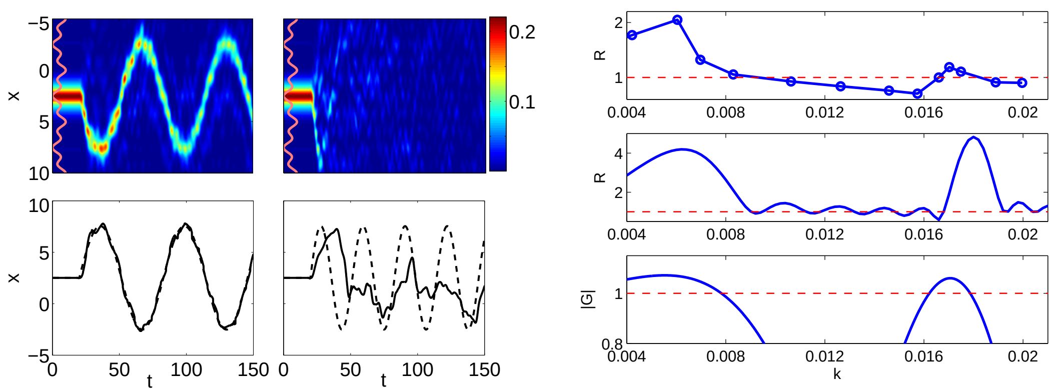

FIG. 1. (Left) Controllable path-following of BEC solitons by temporally modulating one lattice component of the superlattice potential. This can be accomplished for "gap solitons" (depicted) when $g > 0$ and for "bright solitons" when $g < 0$. The top panels depict the spatio-temporal evolution of $|\psi(x,t)|^2$ (the superlattice is shown by the solid pink curve), and the bottom panels show the time-evolution of the soliton's center (solid curve) compared to that of the center of the well in which the soliton is initially placed (dashed curve). Path-following is successful (left panels) provided the superlattice is modulated sufficiently slowly; for superlattices that are modulated too rapidly (right panels), path-following is not successful. (Right) Wavenumber windows with modulational instability for optical beams propagating through a medium consisting of alternating layers of glass and air. The experimental (top), numerical (middle), and analytical (bottom) results match quantitatively (with zero fitting parameters).

NONLINEAR OPTICS: Since my move to Caltech, I have complemented my studies of nonlinear dynamics in BECs by investigating nonlinearity management in optics in collaboration with both theorists (D. J. Frantzeskakis and P. G. Kevrekidis) and experimentalists (M. Centurion, D. Psaltis, and Y. Pu). For light propagation in optical fibers, Eq. (1) (with the independent variable now representing the propagation direction rather than time) is derived as an envelope equation governing the evolution of weakly nonlinear wave packets.

In our initial project [1], we produced the first experimental implementation of nonlinearity management by examining the propagation of light pulses through layers of glass and air (so that the nonlinearity coefficient g is piecewise constant), showing that the blow-up and collapse that would occur in propagation through a uniform medium can be delayed substantially. We corroborated these results with numerical simulations of an NLS equation (which also incorporated losses at the glass-air interfaces) and an analytical solution of an ordinary differential equation description of the system's moments (representing physical quantities such as the beam width, angular momentum, etc.) that used the parameters and initial conditions from the experiment (without any fitting parameters).

We subsequently extended this work by investigating modulational instabilities using a similar setup [2]. (Instabilities of plane waves with respect to modulation arise because the finite intensity of a wave modifies the propagation properties of the medium it is traversing; this leads to localization in position space and the formation of solitary waves.) In our research, which also combined experiments, direct simulations, and theory, we found quantitative agreement in the location of the windows of instability (see the right panel of Fig. 1) obtained in these three manners (again using experimental initial conditions/parameters and zero fitting parameters).

Student Research Projects

In addition to my personal research, my postdoctoral positions have given me the opportunity to advise over twenty undergraduate students, often jointly with other faculty (in mathematics, physics, and biology), to study a wide range of topics. My student projects, which reflect the breadth of my research interests, also serve as springboards to develop collaborations with colleagues and delve into new research areas.

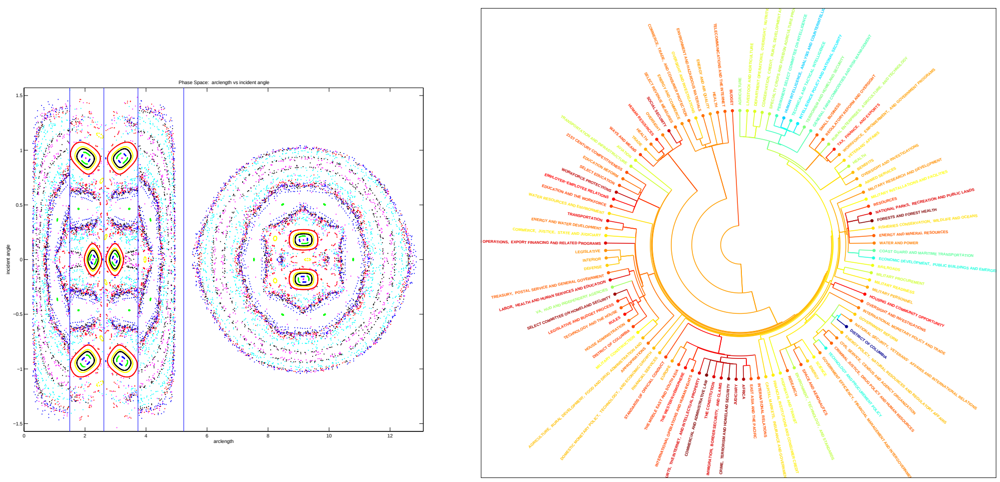

FIG. 2. (Left) Poincaré section of phase space depicting the mixed (regular and chaotic) dynamics of an elliptical mushroom billiard with a triangular stem. The horizontal axis represents the arclength along the billiard boundary, and the vertical axis represents the angle of incidence with respect to the boundary. (Right) Clustering tree ("dendrogram") of the 107th U. S. House of Representatives network, indicating the hierarchies determined by committee and subcommittee assignments. The plot is colored according to partisanship scores computed by applying a singular value decomposition analysis to the roll call votes; the bluest entries represent the most moderate (sub)committees, and the reddest one represent the most partisan ones.

The topics I have advised as student projects include the following:[1]

- Transient Amplification and Contact Line Instabilities in the Spreading of Thin Liquid Films (with S. Peles)
- Dynamics of Plankton Food Chains in the Presence of Seasonal Variation and Fluctuations in Resource Availability (with L. A. Bunimovich and C. Klausmeier)
- Pattern Formation in Periodically Forced Granular Media using Continuum Coupled Maps (with S.-N. Chow)

[1]For a complete list of student projects, see my C. V. Brief descriptions of each project are available at *www.its.caltech.edu/∼mason/research*.

- Community Structure in the United States Congress [17,18] (with P. J. Mucha)
- A Graphical User Interface for Simulating Classical Billiards and its Application to the Study of Elliptical Mushroom Billiards and Few-Particle Billiards [4,5] (with L. A. Bunimovich)
- Quantization of a Free Particle Interacting Linearly with a Harmonic Oscillator

Some of my student projects have developed into full-fledged research projects. I will discuss two of these.

ONE-PARTICLE AND FEW-PARTICLE BILLIARDS: In a mathematical billiard, a confined point particle bounces elastically against a boundary in a (usually two-dimensional) domain, so that the angle of incidence of collisions equals the angle of reflection. The dynamics of billiard systems, which are Hamiltonian, depend on the shape of the boundary. In general, the phase space of a billiard possesses both regular and chaotic regions.

Beginning in Summer 2003, I advised a student project on developing a graphical user interface (GUI) to simulate classical billiard systems. When the GUI was ready in Spring 2004, my student (S. Lansel), L. A. Bunimovich, and I began using it to study the mixed (regular and chaotic) dynamics of billiards whose boundary is shaped like a mushroom with an elliptical cap (a phase portrait of which is shown in Fig. 2a) [5]. This geometry generalizes the recently-studied circular mushroom billiards (which, in turn, generalize stadium billiards), as ellipses have two types of caustics and circles have just one. We also examined two, three, and four finite-size particles colliding elastically in containers [5], demonstrating numerically that few-particle billiards whose corresponding one-particle dynamics are integrable inherit some integrals of motion and exhibit a regular partition of phase space into ergodic components. In Summer 2006, two of my students began extending this work: one studied periodic orbits in (generalized) mushroom billiards and the other studied eigenvalue statistics in quantum mushroom billiards (whose wavefunctions satisfy the Helmholtz equation with homogeneous Dirichlet boundary conditions).

COMMUNITY STRUCTURE IN THE UNITED STATES CONGRESS: With M. E. J. Newman, P. J. Mucha, and two students (C. M. Warmbrand and A. J. Friend), I studied the network of committees and subcommittees in the U. S. House of Representatives [17,18]. In this endeavor, which began as a student project in Summer 2003, the House of Representatives is represented as a bipartite graph with one type of node designating Congressmen and the other designating committees and subcommittees. Edges indicate committee and subcommittee assignments.

We examined the 101st–108th Congresses (1989–2004) and investigated changes in the network structure arising from political events, such as the switch in majority party from Democrats to Republicans in the 1994 election. We focused in particular on the hierarchical organization of cliques ("communities") in the House, finding not only expected connections (between, for example, different subcommittees of the same committee) but also unexpected ones, such as that between the Rules Committee and the Select Committee on Homeland Security in the 107th–108th Houses (see Fig. 2b). We also examined correlations between committee assignments and political viewpoints using a singular value decomposition analysis of voting records.

RESEARCH PLANS

My current research plans are to continue my studies of nonlinear Schrödinger equations and complex networks and to branch out to new projects in applied nonlinear dynamics. To help fund my work, I will be applying for a National Science Foundation applied math grant this fall.

Further Studies of Nonlinear Schrödinger Equations

I have several active projects concerning the application of NLS equations to BECs and optics. Additionally, as NLS equations arise ubiquitously as equations governing the evolution of weakly nonlinear wave packets in various media, there are several additional avenues (for example, in plasma physics and the theory of water waves) along which I can pursue this research.

NONLINEARITY MANAGEMENT: One of the major recent thrusts of the nonlinear waves community has been the application of "management" techniques, in which one or more coefficients of a partial differential equation (PDE) are manipulated algorithmically in time and/or space to simulate its adjustment in experimental settings.

In optics, one can apply *nonlinearity management* by layering materials, so that the nonlinearity coefficient g is piecewise constant as a function of the propagation distance. The excellent agreement between theoretical, numerical, and experimental results obtained in my recent work on wave packet dynamics under nonlinearity management [1,2] suggests a variety of extensions. For example, I plan to analyze the creation and dynamics of solitary waves that result from modulational instabilities. I will also further solidify the mathematical foundations this theoretical framework by deriving an averaged equation for the evolution of solitary waves in the layered medium and compare its dynamics with those of the full NLS equation and the experimental results. I will also investigate competing effects from nonlinearity management and "dispersion management" (in which the coefficient of the Laplacian is adjusted algorithmically), which was originally proposed for optical fibers based on periodically alternating the group-velocity dispersion with opposite signs and was found to be a robust method for supporting breathing solitary waves.

In BECs, one can vary the nonlinearity coefficient $g = g(t)$ in time with an applied magnetic field in the vicinity of a "Feshbach resonance," the study of which is among the most active research topics in experimental and theoretical atomic physics. (Feshbach resonances have provided a key for creating molecular BECs, stabilizing and destabilizing condensates, etc.) Among other phenomena, I plan to study modulational instabilities and the dynamics of solitary and spatially-extended waves when $g(t)$ is alternated periodically and the GP equation has an external periodic (lattice or superlattice) potential. (I recently studied a special case of this framework [9], but there is a lot more to do.) One can similarly employ magnetic fields to make the nonlinearity coefficient spatially inhomogeneous. It is particularly interesting to study spatial periodicity in this framework, so that the system's symmetry is broken in the same way as it would be with a periodic potential. Unlike what happens with a periodic potential, however, the imposed periodicity in this case does not exist in the linear limit, so a band structure of allowed eigenvalues occurs only in the presence of the nonlinearity. I have thus far written one paper on this topic [13], but a full understanding of this setting will require a considerable amount of further research and will entail the development of a theory of "nonlinear band structure" and the investigation of the solitary-wave structures that one can obtain.

PARAMETRIC EXCITATION OF NLS EQUATIONS: By modulating a lattice or superlattice potential periodically in time, one can excite parametric resonances in BECs. To do this, one temporally adjusts the potential's amplitude(s), wavenumber(s), or (for superlattices) the offset of one lattice component with respect to the other. Initially, I plan to complement with analytical studies my prior numerical results [14] indicating that sufficiently slow ("adiabatic") adjustments of a superlattice potential permit one to controllably manipulate solitary waves by transferring them from one lattice well to another, splitting them symmetrically and asymmetrically, etc. I will subsequently study the dynamics and manipulation of solitary waves using various mechanisms of parametric excitation.

VORTICES IN BECs: In two spatial dimensions, BECs can form vortices when they are rotated. One of my current projects (with G. Refael) is to study vortex dynamics in two-component, multi-species BECs. Our first project is to investigate a synchronization phenomenon known as "momentum locking" that we expect to occur between the two components. Because the two atomic species have different masses, this would imply that the resulting vortex lattice includes different numbers of vortices from the two species.

Other Current and Forthcoming Projects

COMMUNITY STRUCTURE IN SOCIAL NETWORKS: A network, or graph, consists of a set of nodes (representing the system's "actors"), with edges determined by some sort of connection between them. Cliques, or *communities*, in a network indicate groups of nodes with quantifiably more "internal" connections (edges between nodes in the group) than "external" connections (edges from nodes in the group to nodes outside it). Real-world social networks possess a modularity that can be quantified in terms of a "community structure" that describes the hierarchies of and relations between its cliques.

One of my primary current projects is to extend my previous research on Congressional committee assignment networks [17,18] to examine the community structure of networks determined by legislation cosponsorship, in which Congressmen are connected by edges to bills. This project, which is in collaboration with mathematician P. J. Mucha, political scientist J. H. Fowler, and two students (A. J. Friend and Y. Zhang), entails not only community detection but also the investigation of techniques to measure the importance of the obtained cliques (which will be compared to known groups such as caucuses and subcommittees) and (in the long term) the development of community-detection procedures that can handle directed graphs (as bills have only one sponsor but myriad cosponsors).

I will concomitantly continue my investigations of committee assignment networks and will also study collegiate social networks, which can be quantified by connections in, e.g., the Facebook data set (which one of my students studied in a Summer 2005 research project).

SPATIO-TEMPORAL CHAOS IN THE COMPLEX GINZBURG-LANDAU EQUATION: The underlying mathematical ubiquity of several spatially extended systems arises because an appropriate (modal) decomposition of the wavefunction satisfying the system's governing equations has Fourier coefficients obeying the same nonlinear PDE, the complex Ginzburg-Landau (CGL) equation.

Because of this, the CGL equation is one of the most important PDEs in applied mathematics. Despite this, its chaotic dynamics is not yet systematically understood. One of my long-term research goals is to use the method of "periodic orbit (cycle) expansions," developed by Predrag Cvitanović and collaborators, to understand the spatio-temporal dynamics of the CGL equation in terms of unstable recurrent patterns.

Over the past twenty years, it has been demonstrated that one can exploit unstable periodic orbits as an effective tool for examining (temporal) chaotic behavior in low-dimensional dynamical systems. Periodic orbit expansions have also been successfully employed to study linear PDEs such as the Schrödinger equation of quantum mechanics. More recently, there have been some attempts to apply this program to nonlinear PDEs such as the Kuramoto-Sivashinsky equation.

The philosophical foundation of this program is that spatio-temporal chaotic dynamics in nonlinear PDEs can be described, in principle, using an infinite hierarchy of unstable spatio-temporal periodic solutions. The mathematical challenge of this physically important program, which is to be applied to progressively more complicated PDEs, lies in showing that what is true in principle can also be applied in practice. Because of the CGL's ubiquity and importance, the next logical step in developing the cycle expansion approach is to apply it to this equation.

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