

A woman in a blue dress is walking away from the camera through a field of many small birds, possibly terns, on a sandy or gravelly ground. The birds are scattered across the field, and the woman is in the center-right of the frame. The background is bright and slightly overexposed.

# United By Noise

Randomness helps swarms stay together

By Kit Yates

# Biblical Proportions

“The locusts came up over all the land of Egypt and settled in all the territory of Egypt; they were very numerous. There had never been so many locusts, nor would there be so many again. For they covered the face of the whole earth, so that the land was darkened; and they did eat every herb of the land, and all the fruit of the trees which the hail had left: and there remained not any green thing in the trees, or in the herbs of the field, through all the land of Egypt. ”  
Exodus(10:14-15)



# Background

## Plagues:

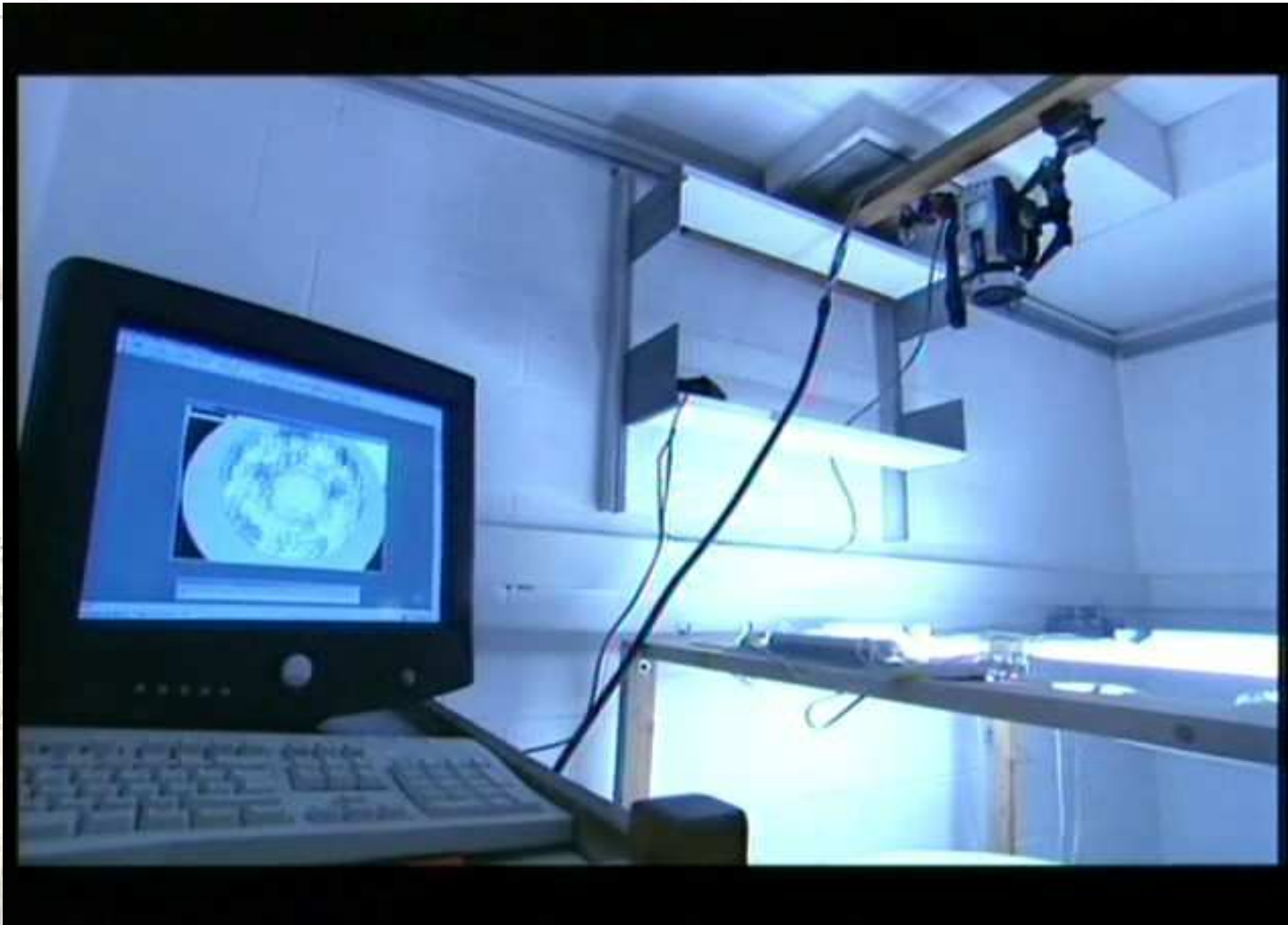
- 60 countries
- 29 million square kilometres
- 20% of the total land surface of the world
- 10% of the world's population



Traffic accidents  
House Eating

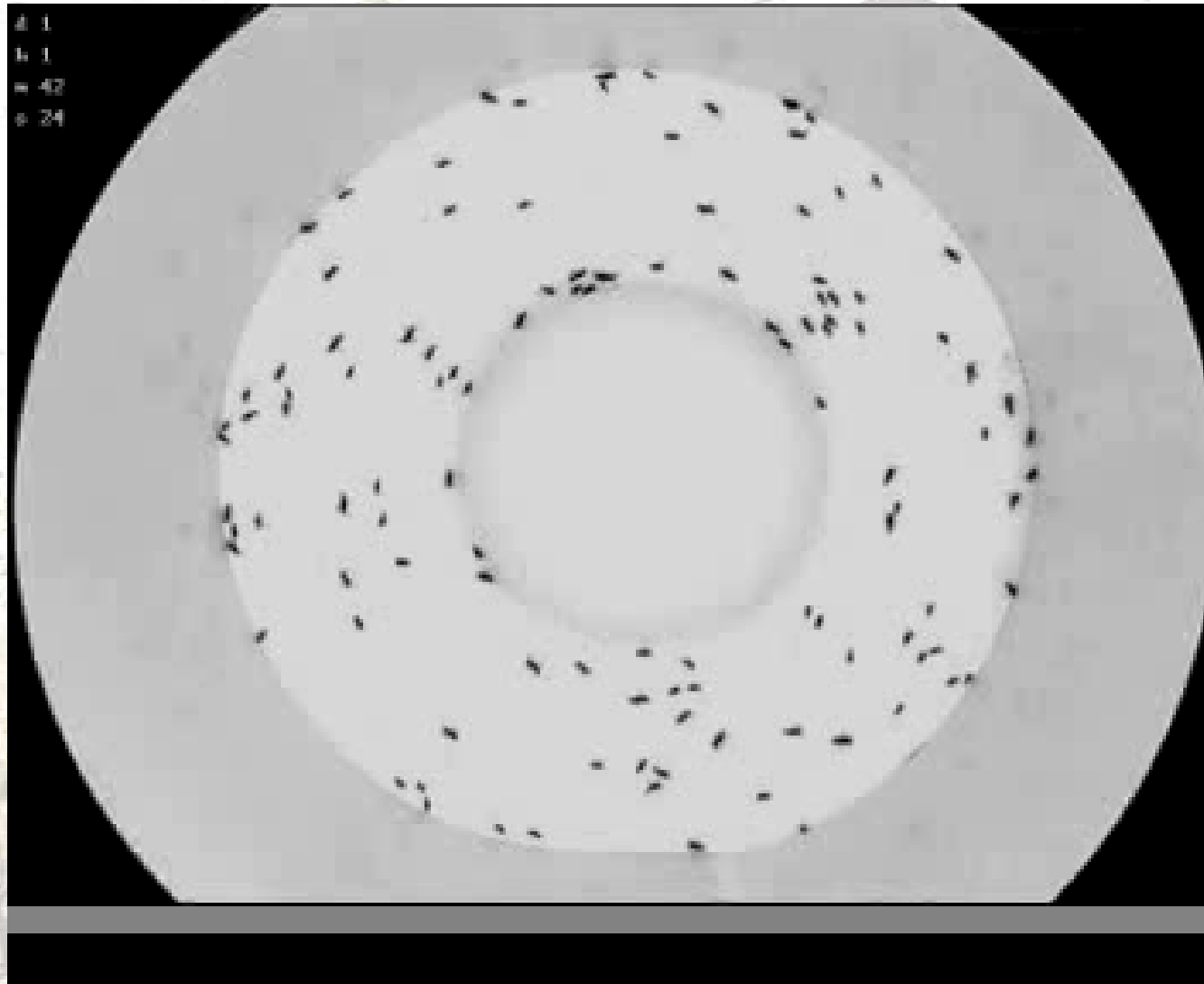


# The Experiments



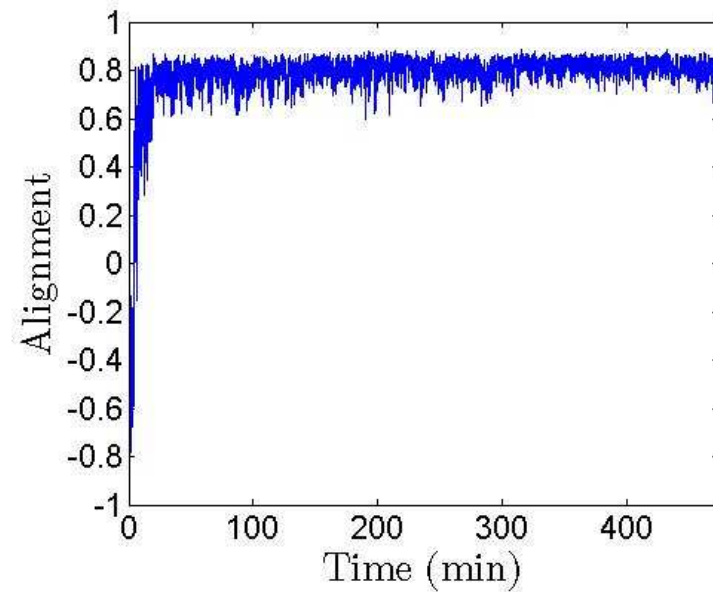
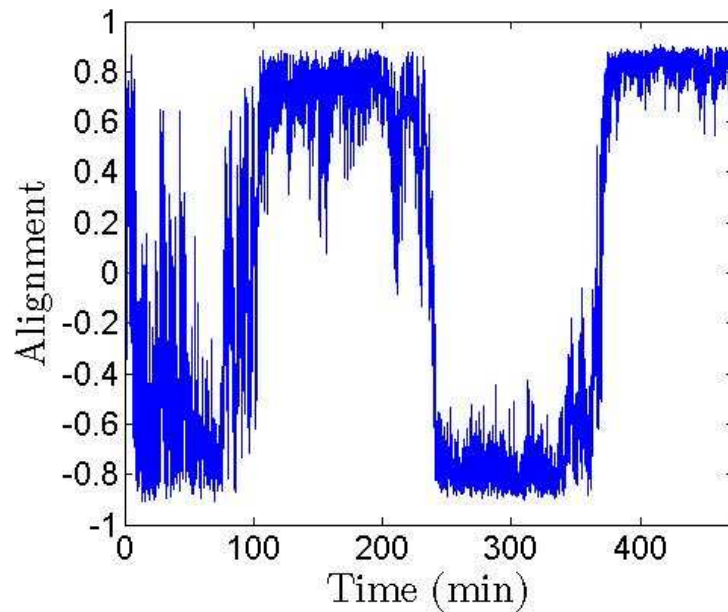
Buhl et al. (2006) From Disorder to Order in Marching Locusts.  
Science, 312:1402-1406.

# Movie



# Experimental Results

- Collective motion
- Switching between two steady states: Clockwise and Anticlockwise
- Noisy/Stochastic process



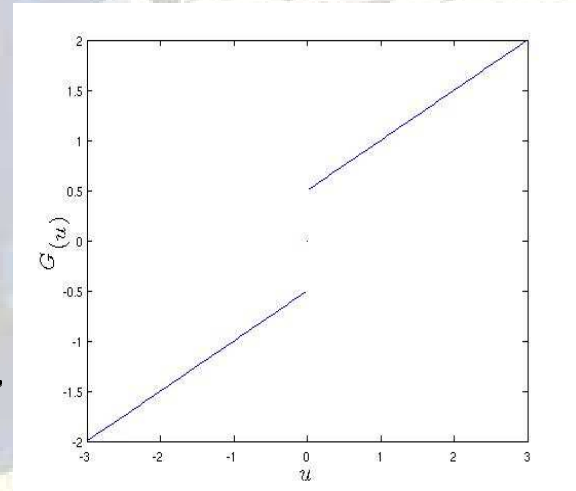
# The model

$$\Delta x_i = u_i \Delta t,$$

$$\Delta u_i = \{G(\bar{u}_i^{loc}) - u_i(t)\} \Delta t + \Delta Q \eta(\bar{u}_i^{loc}),$$

$$\text{where } \bar{u}_i^{loc} = \frac{1}{n_i(t)} \sum_{j \in \mathcal{J}_i^R} u_j(t)$$

$$\text{and } G(z) = \frac{1}{1 + \beta} \{z + \beta \text{sign}(z)\}.$$



# Stochastic Differential Equations (SDEs)

## Typical SDE

$$U(t + dt) = U(t) + F(U(t))dt + \sqrt{2D(U(t))}dW$$

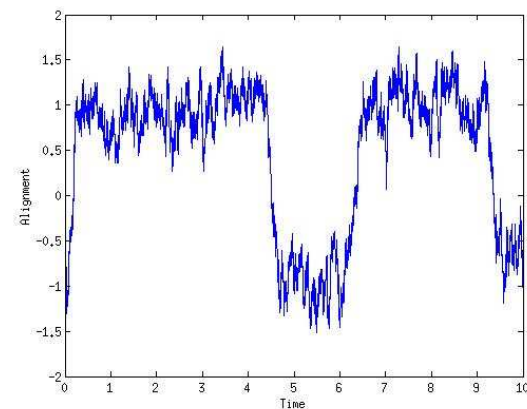
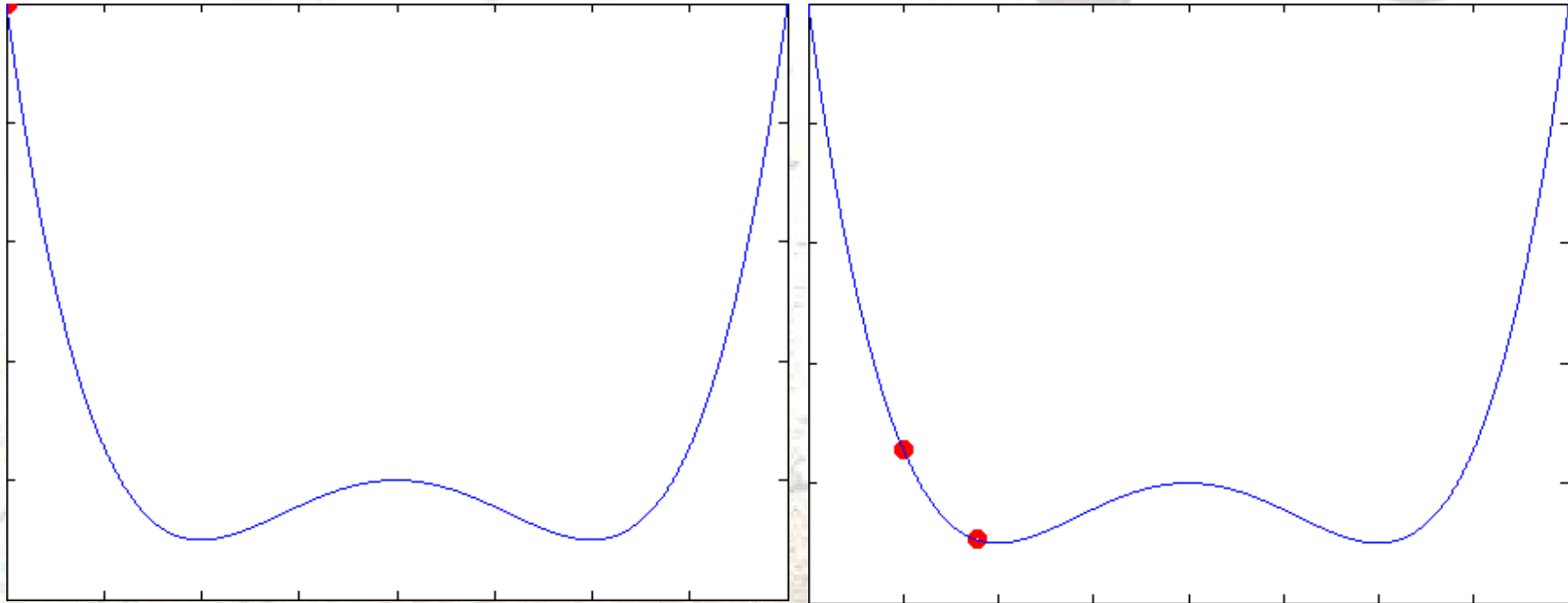
drift,  $F$ , diffusion  $D$  and  $dW \sim \sqrt{dt}N(0, 1)$ ,  
the standard Wiener process.

Find  $F$  and  $D$  then find potential  $\phi$ :

$$\phi(U) = - \int_0^U \frac{F(s)}{D(s)} ds + \ln(D(U)).$$



# The Difference Between an ODE and an SDE



# Equation-Free Analysis

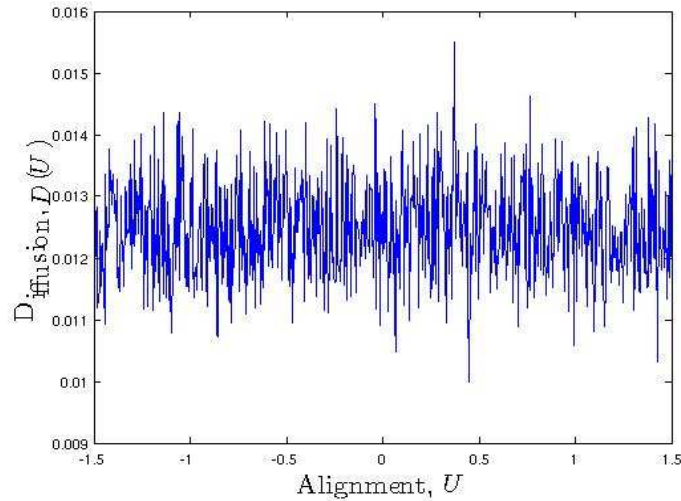
$$F(U) \approx \left\langle \frac{U(t+dt) - U(t)}{dt} \right\rangle,$$

$$D(U) \approx \frac{1}{2} \left\langle \frac{[U(t+dt) - U(t)]^2}{dt} \right\rangle,$$

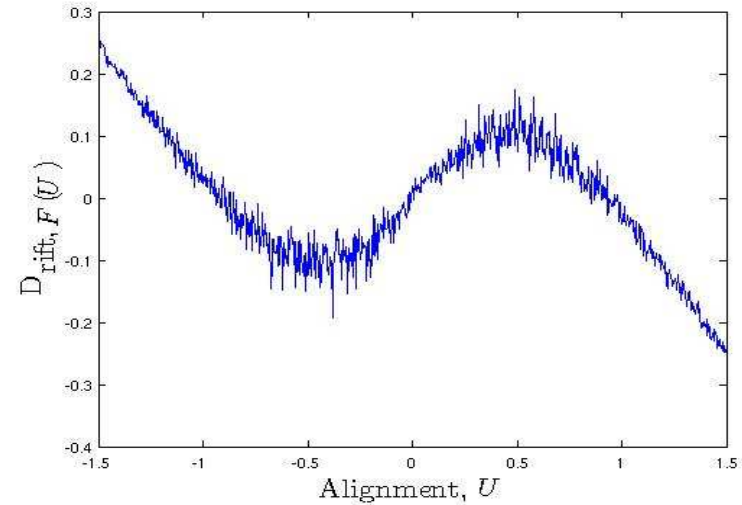
Recall the SDE:

$$U(t+dt) - U(t) = F(U(t))dt + \sqrt{2D(U(t))}dW$$

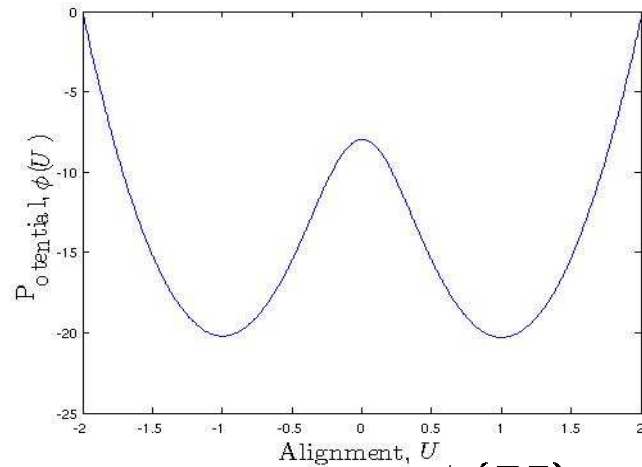
# Model Results



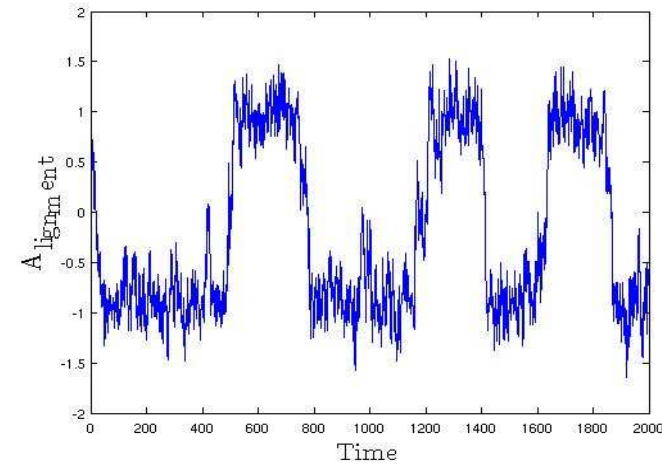
Diffusion,  $D(U)$



Drift,  $F(U)$

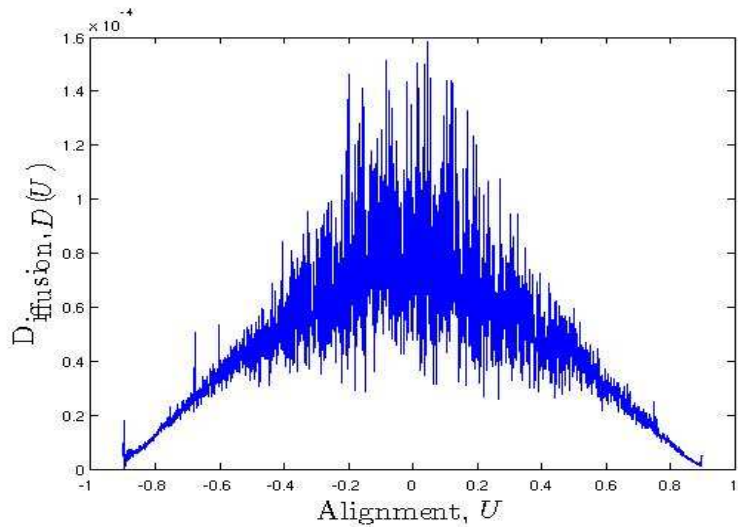


Potential  $\phi(U)$

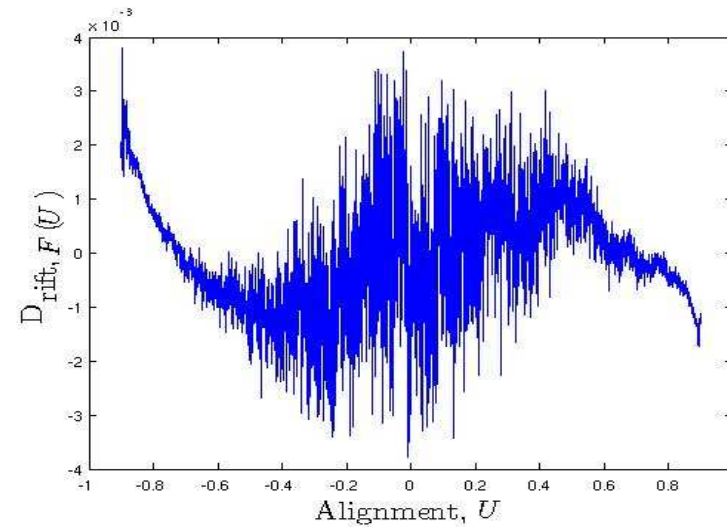


Alignment Time Series <sup>11</sup>

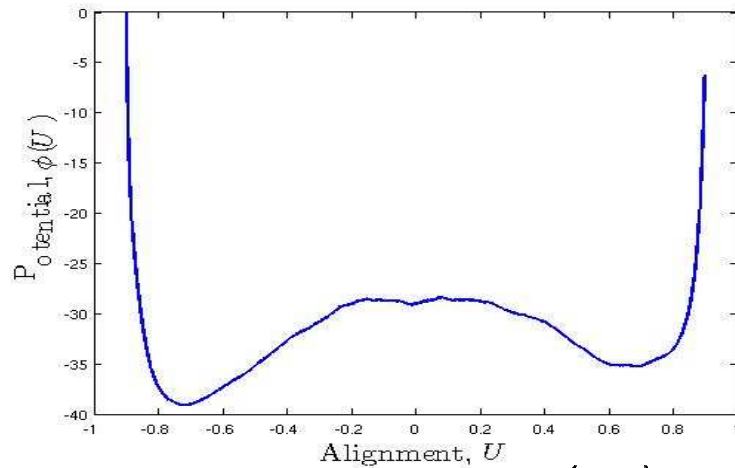
# Data Analysis



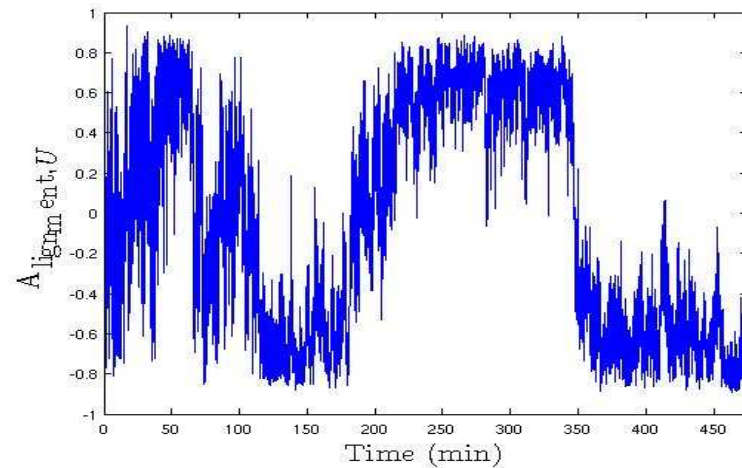
Diffusion,  $D(U)$



Drift,  $F(U)$



Potential  $\phi(U)$



Alignment Time Series <sup>12</sup>

# Model Revision

Recall the velocity update equation,

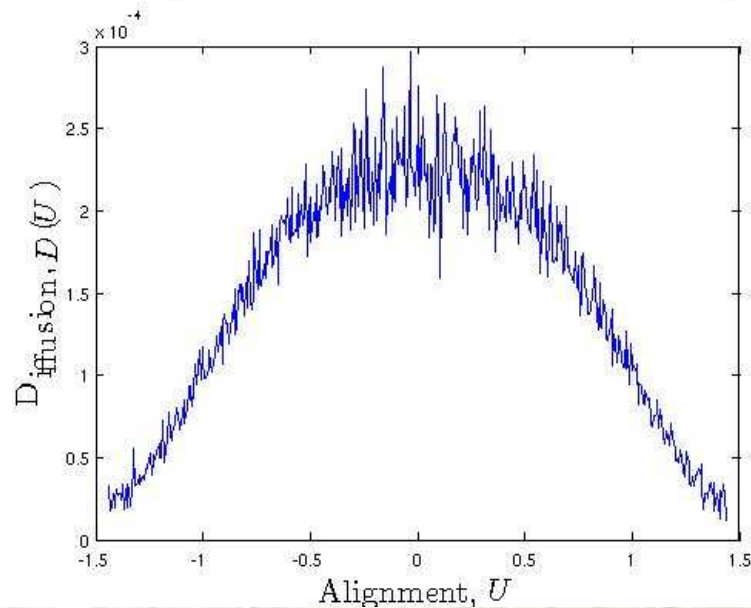
$$\Delta u_i = \{G(\bar{u}_i^{loc}) - u_i(t)\} \Delta t + \Delta Q \eta(\bar{u}_i^{loc}).$$

Previously  $\eta(\bar{u}_i^{loc}) \equiv 1$ .

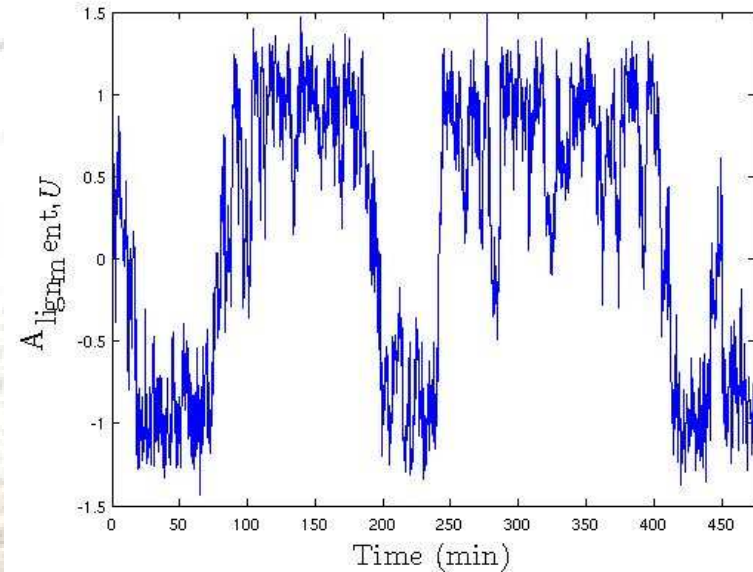
We now make  $\eta(\bar{u}_i^{loc}) = \frac{3}{2} \left\{ 1 - \left( \frac{\bar{u}_i^{loc}}{|\bar{u}_i^{loc}|_{max}} \right)^2 \right\}$ ,

where  $|\bar{u}_i^{loc}|_{max}$  is the maximum of the absolute value of the mean local velocity.

# Revised Model Results

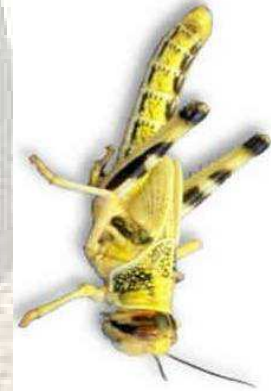
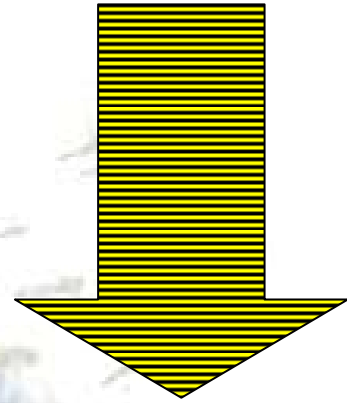
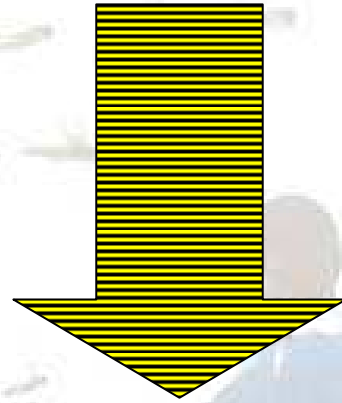
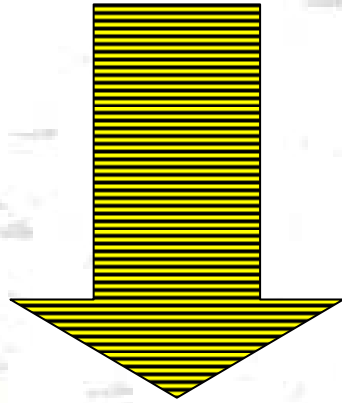
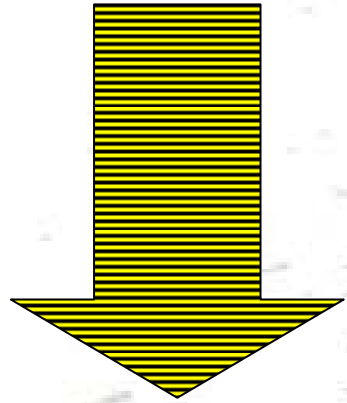


Diffusion,  
 $D(U)$



Alignment Time  
Series

- When group becomes unaligned individuals increase the randomness of their motion.



Bazazis et al. (2008) Collective Motion and Cannibalism in Locust Migratory Bands  
Current Biology 18, 735–739.

# Conclusions

## Explanations

- Evolution - Alignment increase harvesting efficiency and reduce predation
- Cannibalism – Dangerous to fall out of line as sides are more vulnerable

**Increased individual randomness  
when unaligned help the swarm  
stay together.**

For more details see: Yates et al. (2009) Inherent noise facilitates cohesive behaviour in swarms. Under Review, PNAS.



# Acknowledgement

- Supervisors: Radek Erban, David Sumpter and Carlos Escudero.
- Experimentalist and Colaborators: Iain Couzin, Jerome Buhl, Yannis Keverekidis and Philip Maini.
- The Doctoral Training Centre.
- EPSRC/BBSRC.
- Oxford SIAM Student Chapter Committee.