Electric–Magnetic Duality and the Dualized Standard Model

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Notations

- $X =$ Minkowski space with signature $+---$
- $\mu, \nu, \ldots =$ spacetime indices $=$ 0,1,2,3
- $i, j, \ldots =$ spatial indices or group indices
- repeated indices are summed
- $G =$ gauge group $=$ compact, connected Lie group (usually $U(n), SU(n), O(n)$)

Conventions

- Maxwell theory = theory of electromagnetism = abelian theory
- Yang–Mills theory = nonabelian (gauge) theory
- Spacetime = Minkowski space
- Functions are continuous or smooth
- Manifolds are C^∞
- $\hbar = 1, c = 1$

Dictionary

base space	$\longleftrightarrow$	spacetime
structure group	$\longleftrightarrow$	gauge group
principal bundle	$\longleftrightarrow$	gauge theory
principal coordinate bundle	$\longleftrightarrow$	gauge theory in a particular gauge
connection	$\longleftrightarrow$	gauge potential
curvature	$\longleftrightarrow$	gauge field
holonomy	$\longleftrightarrow$	phase factor
bundle reduction	$\longleftrightarrow$	symmetry breaking
section $\sigma: X \to E$	$\longleftrightarrow$	Higgs fields

Lecture 1

Electric–magnetic duality

Gauge invariance

Consider an electrically charged particle in an electromagnetic field:

- Described by a wavefunction $\psi(x)$
- $\psi(x)$ not measurable, only $|\psi(x)|^2$
- Freedom in redefining phase $\rightsquigarrow$ gauge symmetry
- Yang and Mills generalized this phase freedom to an arbitrary element of a Lie group

How can we compare the phases at neighbouring points in spacetime?
How can we 'parallelly propagate' the phase?
Answer: we can if given a potential $A_\mu(x)$

Potential transforms as ($S(x) \in G$)

$$A_\mu(x) \mapsto S(x)\, A_\mu(x)\, S^{-1}(x) - (\tfrac{i}{g})\, \partial_\mu S(x)\, S^{-1}(x)$$

Gauge variables

Introduce gauge field
$F_{\mu\nu}(x) = \partial_\nu A_\mu(x) - \partial_\mu A_\nu(x) + ig[A_\mu(x), A_\nu(x)]$

In *classical* electromagnetism there is no need to introduce the potential.

The Bohm–Aharonov experiment demonstrates that the potential is necessary to describe the motion of a *quantum* particle (e.g. an electron) in an electromagnetic field.

This really vindicates the geometric description of gauge theory we have now.

Yang has proved that it is the set of variables comprising the holonomy of loops which describes a gauge theory exactly:

$$\Phi(C) = P \exp ig \int_C A_\mu(x)$$

Gauge group

In building a physical theory, we must look among experimental facts to collect our ingredients. The potential fixes only the Lie algebra. To select out from among the locally isomorphic ones the correct Lie group we must look at the particle spectrum, that is, what kind of and how many particles exist or are postulated to exist.

In electromagnetism all charges are multiples (in fact, just ± 1) of a fundamental charge e, so that wavefunctions transform as

$$\psi \mapsto e^{\pm ie\Lambda}\psi,$$

we can parametrize the circle group $U(1)$ corresponding to the phase by $[0, 2\pi/e]$. In fact, *charge quantization* is equivalent to having $U(1)$ as the gauge group of electromagnetism.

Gauge group

For pure electromagnetism without charges, the only relevant gauge transformation are those of A_μ:

$$A_\mu \mapsto A_\mu + \partial_\mu \Lambda,$$

so that the group will just be the real line given by the scalar function $\Lambda(x)$.

Similarly for Yang–Mills theory, e.g. $\mathfrak{su}(2)$. If it contains particles with a 2-component wave function $\psi = \{\psi_i,\ i = 1, 2\}$, then

$$\psi \mapsto S\psi, \quad S \in SU(2),$$

so that the effect of S and $-S$ are not identical. Hence gauge group is $SU(2)$. If there are no charges then the effects on A_μ of S and $-S$ are identical

$$A_\mu \mapsto S\, A_\mu\, S^{-1} - \frac{i}{g} \partial_\mu S\, S^{-1}$$

Hence gauge group is $SO(3)$.

Sources and monopoles

For the moment we wish to distinguish between two types of charged particles: sources and monopoles.

In a pure gauge theory, we have Yang–Mill's equation:

$$D_\nu F^{\mu\nu} = 0.$$

Electric sources are those particles that give rise to a nonvanishing right hand side of the above equation:

$$D_\nu F^{\mu\nu} = -j^\mu, \quad j^\mu = g\bar{\psi}\gamma^\mu\psi.$$

Magnetic monopoles are topological in nature and are represented geometrically by nontrivial G-bundles. They are classified by elements of $\pi_1(G)$.

However, in view of the electric–magnetic duality we shall study, the concepts of 'electric' and 'magnetic' are interchangeable depending on which description one uses.

Abelian duality

$$ {}^*F^{\mu\nu} = -\tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma} $$

Classical Maxwell theory is invariant under duality:

$$\begin{aligned} \partial_\nu F^{\mu\nu} &= 0 \quad [\mathrm{d}\,{}^*F = 0] \\ \partial_\nu {}^*F^{\mu\nu} &= 0 \quad [\mathrm{d}\,F = 0] \end{aligned}$$

Poincaré lemma:

$$\begin{aligned} F_{\mu\nu}(x) &= \partial_\nu A_\mu(x) - \partial_\mu A_\nu(x), \\ {}^*F_{\mu\nu}(x) &= \partial_\nu \tilde{A}_\mu(x) - \partial_\mu \tilde{A}_\nu(x). \end{aligned}$$

The two potentials transform independently:

$$\begin{aligned} A_\mu(x) &\longrightarrow A_\mu(x) + \partial_\mu \Lambda(x), \\ \tilde{A}_\mu(x) &\longrightarrow \tilde{A}_\mu(x) + \partial_\mu \tilde{\Lambda}(x). \end{aligned}$$

Abelian duality

This means that the full symmetry of this theory is doubled to $U(1) \times \tilde{U}(1)$.

This dual symmetry means that what we call 'electric' or 'magnetic' is entirely a matter of choice.

In the presence of electric charges:

$$\begin{aligned} \partial_\nu F^{\mu\nu} &= -j^\mu \\ \partial_\nu {}^*F^{\mu\nu} &= 0. \end{aligned}$$

Alternatively:

$$\begin{aligned} \partial_\nu F^{\mu\nu} &= 0 \\ \partial_\nu {}^*F^{\mu\nu} &= -\tilde{j}^\mu. \end{aligned}$$

If both types of charges existed in nature:

$$\begin{aligned} \partial_\nu F^{\mu\nu} &= -j^\mu \\ \partial_\nu {}^*F^{\mu\nu} &= -\tilde{j}^\mu. \end{aligned}$$

Wu–Yang criterion

The free Maxwell action is:

$$\mathcal{A}_F^0 = -\tfrac{1}{4}\int F_{\mu\nu}F^{\mu\nu}.$$

he true variables of the theory as we said before are the A_μ, so we should put in a constraint to say that $F_{\mu\nu}$ is the curl of A_μ:

$$\mathcal{A} = \mathcal{A}_F^0 + \int \lambda_\mu(\partial_\nu {}^*F^{\mu\nu}).$$

Varying w.r.t. $F_{\mu\nu}$, we obtain $\partial_\nu F^{\mu\nu} = 0$.

Dually, start with

$$\mathcal{A}_F^0 = \tfrac{1}{4}\int {}^*F_{\mu\nu}{}^*F^{\mu\nu},\ \mathcal{A} = \mathcal{A}_F^0 + \int \tilde{\lambda}_\mu(\partial_\nu F^{\mu\nu}).$$

and obtain $\partial_\nu {}^*F^{\mu\nu} = 0$.

Wu–Yang criterion

This method applies to the interaction of charges and fields as well. Start with the free field plus free particle action:

$$\mathcal{A}^0 = \mathcal{A}_F^0 + \int \bar{\psi}(i\partial_\mu\gamma^\mu - m)\psi,$$

add monopole constraint

$$\mathcal{A}' = \mathcal{A}^0 + \int \lambda_\mu(\partial_\nu {}^*F^{\mu\nu} + \tilde{\jmath}^\mu).$$

Obtain full set of equations of motion:

$$\begin{aligned} \partial_\nu F^{\mu\nu} &= 0 \\ \partial_\nu {}^*F^{\mu\nu} &= -\tilde{\jmath}^\mu \\ (i\partial_\mu\gamma^\mu - m)\psi &= -\tilde{e}\tilde{A}_\mu\gamma^\mu\psi \end{aligned}$$

Nonabelian duality?

We would of course like to generalize this duality to the nonabelian Yang–Mills case.

First of all, despite appearances the Yang–Mills equation

$$D_\nu F^{\mu\nu} = 0$$

and the Bianchi identity

$$D_\nu {}^*F^{\mu\nu} = 0$$

are not dual-symmetric, because the correct dual of the Yang–Mills equation ought to be

$$\tilde{D}_\nu {}^*F^{\mu\nu} = 0,$$

where $\tilde{D}_\nu$ is the covariant derivative corresponding to a dual potential.

Nonabelian duality?

Secondly, the Yang–Mills equation, unlike its abelian counterpart, says nothing about whether the 2-form ${}^*F$ is closed or not. Nor is the relation

$$F_{\mu\nu}(x) = \partial_\nu A_\mu(x) - \partial_\mu A_\nu(x) + ig[A_\mu(x), A_\nu(x)]$$

about exactness at all.

In other words, Yang–Mills equation does not guarantee the existence of a dual potential, in contrast to the Maxwell case.

In fact, Gu and Yang have constructed a counter-example.

Because the true variables of a gauge theory are the potentials and not the fields, this means that Yang–Mills theory is *not symmetric* under the Hodge star operation.

Generalized duality

Nevertheless, electric–magnetic duality is a very useful physical concept. So the natural step is to seek a more general duality transform (˜) satisfying the following properties:

1. $(\quad)^{\sim\sim} = \pm(\quad)$,
2. electric field $F_{\mu\nu} \stackrel{\sim}{\longleftrightarrow}$ magnetic field $\tilde{F}_{\mu\nu}$,
3. both A_μ and $\tilde{A}_\mu$ exist as potentials (away from charges),
4. magnetic charges are monopoles of A_μ, and electric charges are monopoles of $\tilde{A}_\mu$,
5. ˜ reduces to * in the abelian case.

Generalized duality

$$\begin{tikzcd}
\boxed{\begin{array}{c} A_\mu \text{ exists as} \\ \text{potential for } F_{\mu\nu} \\ (F = \mathrm{d}\, A) \end{array}} \arrow[r, Leftrightarrow, "\text{Poincaré}"] \arrow[d, Leftrightarrow] & \boxed{\begin{array}{c} \text{Defining constraint} \\ \partial_\mu {}^*F^{\mu\nu} = 0 \\ (\mathrm{d}\, F = 0) \end{array}} \arrow[d, Leftrightarrow, "\text{Gauss}"] \\
\boxed{\begin{array}{c} \text{Principal } A_\mu \\ \text{bundle trivial} \end{array}} & \boxed{\begin{array}{c} \text{No magnetic} \\ \text{monopole } \tilde{e} \end{array}} \\
\text{GEOMETRY} & \text{PHYSICS}
\end{tikzcd}$$

Loop variables

Recall that we define the Dirac phase factor $\Phi(C)$ of a loop C

$$\Phi[\xi] = P_s \exp ig \int_0^{2\pi} ds\, A_\mu(\xi(s))\dot{\xi}^\mu(s),$$

where we parametrize the loop C:

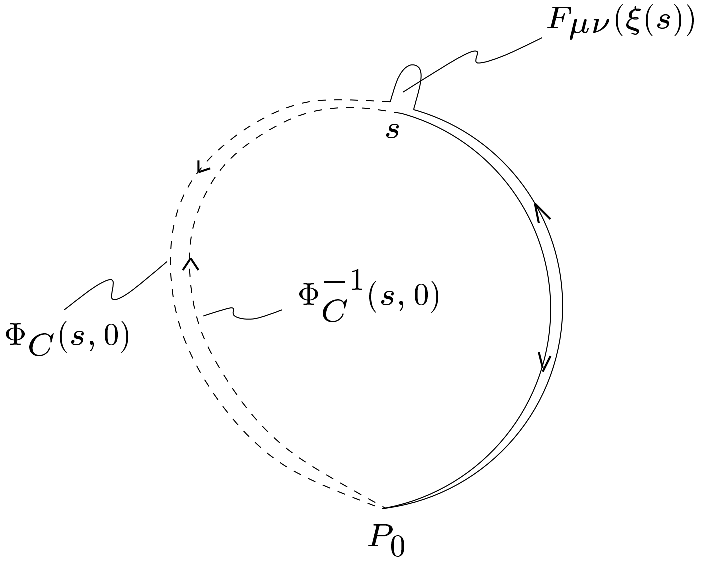
$$C: \quad \{\xi^\mu(s): s = 0 \to 2\pi,\ \xi(0) = \xi(2\pi) = \xi_0\},$$

and a dot denotes differentiation with respect to the parameter s.

Loop variables

$$F_\mu[\xi|s] = \frac{i}{g}\Phi^{-1}[\xi]\,\delta_\mu(s)\Phi[\xi]$$

$$G_{\mu\nu}[\xi|s] = \delta_\nu(s)F_\mu[\xi|s] - \delta_\mu(s)F_\nu[\xi|s] + ig[F_\mu[\xi|s], F_\nu[\xi|s]]$$



Loop variables

The action:

$$\mathcal{A}_F^0 = -\frac{1}{4\pi\bar{N}} \int \delta\xi \int_0^{2\pi} ds \, \mathrm{Tr}\{F_\mu[\xi|s] F^\mu[\xi|s]\} |\dot{\xi}(s)|^{-2}$$

In pure Yang–Mills theory, the constraint:

$$G_{\mu\nu}[\xi|s] = 0$$

Wu–Yang criterion gives Polyakov equation

$$\delta_\mu(s) F^\mu[\xi|s] = 0.$$

In the presence of a monopole charge –

$$G_{\mu\nu}[\xi|s] = -J_{\mu\nu}[\xi|s]$$

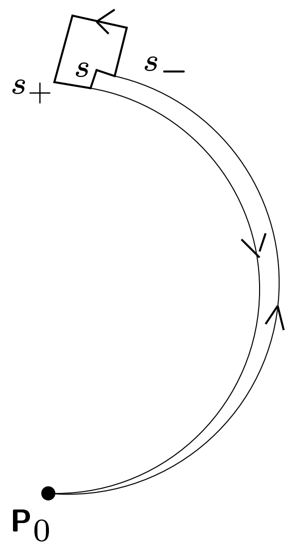
Nonabelian duality

New variables:

$$E_\mu[\xi|s] = \Phi_\xi(s,0) F_\mu[\xi|s] \Phi_\xi^{-1}(s,0)$$

Equations become

$$\begin{aligned} \delta_\nu(s) E_\mu[\xi|s] - \delta_\mu(s) E_\nu[\xi|s] &= 0 \\ \delta^\mu(s) E_\mu[\xi|s] &= 0 \end{aligned}$$



Nonabelian duality

Lecture 2

Some questions in present-day theoretical particle physics

Standard Model: spectrum

Vector bosons (also known as gauge bosons):
γ; W^+, W^-, Z^0; g

(photon; massive vector bosons; gluons)

Quarks: t, b; c, s; u, d

(top, bottom; charm, strange; up, down)

Leptons: τ, ν_τ; μ, ν_μ; e, ν_e

(tauon, tau neutrino; muon, muon neutrino; electron, electron neutrino)

In a full quantum theory, these particles all have corresponding antiparticles

Postulated: scalars called Higgs

Standard Model: symmetries

$$SU(3) \times SU(2) \times U(1)/\mathbb{Z}_6$$

Everything occurs 3 times: 3 generations

Very similar properties except for masses e.g.:

$$m_\tau \colon m_\mu \colon m_e \cong 3000 \colon 200 \colon 1$$

U triad in generation space not aligned with D triad $\rightsquigarrow$ mixing (CKM for quarks, MNS for leptons)

Symmetry breaking

Physical idea: the action is invariant under the action of the gauge group, but the vacuum is invariant under only a proper subgroup

Strong interaction: $SU(3)$ exact

Electroweak interaction: $U(2)$ broken

$\mathfrak{su}(2) \oplus \mathfrak{u}(1)$ has generators T_0, T_1, T_2, T_3

Symmetry breaking effected by extra term in Yang–Mills action

$$\mathcal{A}_H = \int D_\mu \phi D^\mu \phi + V(\phi)$$

with

$$V(\phi) = -\tfrac{\mu^2}{2}|\phi|^2 - \tfrac{\lambda}{4}|\phi|^4 \quad (\lambda > 0)$$

and

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

Symmetry breaking

Covariant derivative D_μ will contain four gauge components: $W_\mu^1, W_\mu^2, W_\mu^3$ corresponding to the $\mathfrak{su}(2)$ part with coupling g_2, and Y_μ to the $\mathfrak{u}(1)$ part with coupling g_1.

If $\mu^2 < 0$, the vacuum (with $V(\phi)$ minimum) is given by

$$|\phi_0| = -\mu^2/\lambda = \eta \neq 0$$

Now choose a gauge

$$\phi_0 = \eta \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Thus, the vacuum corresponds to a particular direction in the space of $\mathfrak{su}(2) \oplus \mathfrak{u}(1)$ and once this choice is made, the physics will no longer be invariant under the whole of the $U(2)$ group.

Symmetry breaking

ϕ complex means there will be a phase rotation left over after fixing a direction as above, and it is this 'little group' $U(1)$ that is identified as the abelian electromagnetic group we studied before.

Geometrically the group $U(2)$ is a torus $S^3 \times S^1$, and the residual symmetry group is a 'diagonal' $U(1)$ of this torus, generated by a linear combination of T_0 and T_3 as shown below.

Quantum excitations give rise to a new scalar field σ:

$$\phi(x) = \begin{pmatrix} 0 \\ \eta + \frac{\sigma(x)}{\sqrt{2}} \end{pmatrix}$$

Symmetry breaking

Define now the Weinberg angle

$$\sin\theta_W = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}$$

and new fields

$$\begin{aligned} A_\mu &= -\sin\theta_W\, W_\mu^3 + \cos\theta_W\, Y_\mu, \\ Z_\mu &= \cos\theta_W\, W_\mu^3 + \sin\theta_W\, Y_\mu, \end{aligned}$$

Can re-write the action $\mathcal{A}_F^0 + \mathcal{A}_H$ in terms of the new fields $\sigma, W_\mu^1, W_\mu^2, Z_\mu, A_\mu$.

Compare with Klein–Gordon lagrangian

$$-\partial_\mu\phi\partial^\mu\phi - m^2\phi^2$$

can identify the massive fields $\sigma, W_\mu^1, W_\mu^2, Z_\mu,$ while the field A_μ which remains massless we can identify as the electromagnetic field.

Symmetry breaking with fermions

Introduce further terms in the action

$$\mathcal{A}_L = \int \bar{\psi} D_\mu \gamma^\mu \psi + \int \rho \bar{\psi}_L \phi \psi_R + h.c.$$

With only one generation (e and ν_e)

$$\psi_L = \left(\begin{array}{c} \nu_e \\ e \end{array} \right)_L, \quad \psi_R = e_R, \quad \psi = \psi_L + \psi_R$$

with $e_L = \frac{1}{2}(1+\gamma_5)e, e_R = \frac{1}{2}(1-\gamma_5)e$ and ν_e purely left-handed.

Compare with Dirac langrangian

$$\bar{\psi}(i\partial_\mu \gamma^\mu - m)\psi$$

conclude that the electron acquires a mass through the Higgs field ϕ in the Yukawa term. The purely left-handed neutrino remains massless in this formulation.

Fermion mass matrices

Symmetry breaking via Higgs mechanism is only way for particles to acquire mass.

By confronting the three component groups $SU(3)$, $SU(2)$, and $U(1)$ of the Standard Model with what is observed (or desired to be observed), we have the following situation as regards the mass.

Of the gauge bosons, the 8 gluons of colour $SU(3)$ are massless, so is the particular generator of $SU(2) \times U(1)$ identified as the photon. The 3 remaining gauge bosons are massive.

Of the fermions (which are the charges), both the quarks and the charged leptons acquire mass through Yukawa terms involving the Higgs field.

There is no *theoretical* reason to demand that the neutrinos are massless, and indeed they most probably have a small mass.

Running mass matrices

Quantum field theory as presently formulated can only yield measurable quantities by a perturbative calculation, and the only realistic way to do so is by summing Feynman diagrams.

Even putting aside the question of ghost terms for a nonabelian gauge theory, we are immediately faced with two problems.

Firstly each individual Feynman diagram usually contains divergent integrals.

And even after regularizing these integrals one has to make sure that the perturbative series can be sensibly summed.

These issues are dealt with under the heading of 'renormalization'.

The renormalization procedure introduces a scale dependence on the physical quantities in the theory. Example: 'running coupling constant'.

Running mass matrices

The dependence on scale t of any given quantity, such as the mass matrix, is explicitly known, via the relevant 'renormalization group equation'. For example, for the quarks we have:

$$\begin{aligned} \frac{dU}{dt} &= \frac{3}{32\pi^2}(UU^\dagger - DD^\dagger)U + (\Sigma_u - A_u)U \\ \frac{dD}{dt} &= \frac{3}{32\pi^2}(DD^\dagger - UU^\dagger)D + (\Sigma_d - A_d)D \end{aligned}$$

For a mass matrix with both eigenvalues and eigenvectors depending on scale, it is not obvious how one can define the physical mass and the physical state vector.

In the next lecture we shall make a proposal for doing so in the Dualized Standard Model.

’t Hooft’s theorem and duality

There is an unexplained lopsidedness about the SM: exact colour with confined charges but broken electroweak with free charges.

’t Hooft introduces two loop operators:

- $A(C)$ measures the magnetic flux through C and creates electric flux along C
- $B(C)$ measures the electric flux through C and creates magnetic flux along C

They satisfy the commutation relation

$$A(C)B(C') = B(C')A(C)\exp(2\pi i n/N)$$

He defined only

$$A(C) = \mathrm{tr}\,\Phi(C)$$

’t Hooft’s theorem and duality

These two operators play dual roles in the sense we have been considering in the first lecture. However, there was no “magnetic” potential available at the time, so that the definition of $B(C')$ was not explicit, only through the commutation relation above.

But we have now in fact constructed the magnetic potential $\tilde{A}_\mu$, and we can prove the commutation relation, so that we know that our duality is the same as ’t Hooft’s.

This also means that we can apply the following result to the duality we find.

’t Hooft’s Theorem. *If the Wilson loop operator of an $SU(N)$ theory and its dual theory satisfy the commutation relation given above, then:*

$$\begin{aligned} SU(N) \text{ confined} \quad &\Longleftrightarrow \quad \widetilde{SU(N)} \text{ broken} \\ SU(N) \text{ broken} \quad &\Longleftrightarrow \quad \widetilde{SU(N)} \text{ confined} \end{aligned}$$

Lecture 3

Dualized Standard Model

Dualized Standard Model

THEORY

- $SU(N) \rightsquigarrow SU(N) \times \widetilde{SU(N)}$
- $SU(N)$ confined $\Longleftrightarrow \widetilde{SU(N)}$ broken

EXPERIMENT

- $SU(3)$ colour is confined
- $\exists$ 3 generations of fermions, very similar except for masses

DSM MAIN ASSUMPTION

- Generation symmetry $=$ dual colour

Questions in SM

- 3 generations
- mass hierarchy
- fermion mixing
- origin of Higgs

Higgs fields

RECALL: dual transform involves local rotation matrices $\omega(x)$ relating the two gauge frames

Rows transform as $\bar{\mathbf{3}}$ of colour

Columns transform as $\mathbf{3}$ of dual colour

Promote to fields: 3 triplets $\phi_a^{(a)}$

$(a) = 1, 2, 3$ label the 3 triplets

$a = 1, 2, 3$ label their 3 dual colour components

Higgs fields

Introduce potential to break $\widetilde{SU}(3)$ completely:

$$
\begin{aligned}
V[\phi] \quad &= \quad -\mu \sum_{(a)} |\phi^{(a)}|^2 + \lambda \left\{ \sum_{(a)} |\phi^{(a)}|^2 \right\}^2 \\
&\quad + \kappa \sum_{(a)\neq(b)} |\bar{\phi}^{(a)}.\phi^{(b)}|^2
\end{aligned}
$$

Vacuum given by

$$
\phi^{(1)} = \zeta \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}, \; \phi^{(2)} = \zeta \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix}, \; \phi^{(3)} = \zeta \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix}
$$

with

$$
x^2 + y^2 + z^2 = 1, \quad \zeta = \sqrt{\mu/2\lambda}
$$

Mass matrix

Fermions acquire mass through Yukawa terms in Lagrangian

$$m = m_T \begin{pmatrix} x \\ y \\ z \end{pmatrix} (x\ y\ z) = m_T \begin{pmatrix} x^2 & xy & xz \\ xy & y^2 & yz \\ xz & yz & z^2 \end{pmatrix}$$

where the fermion type $T = U, D, L, N$.

Renormalization Group Equation

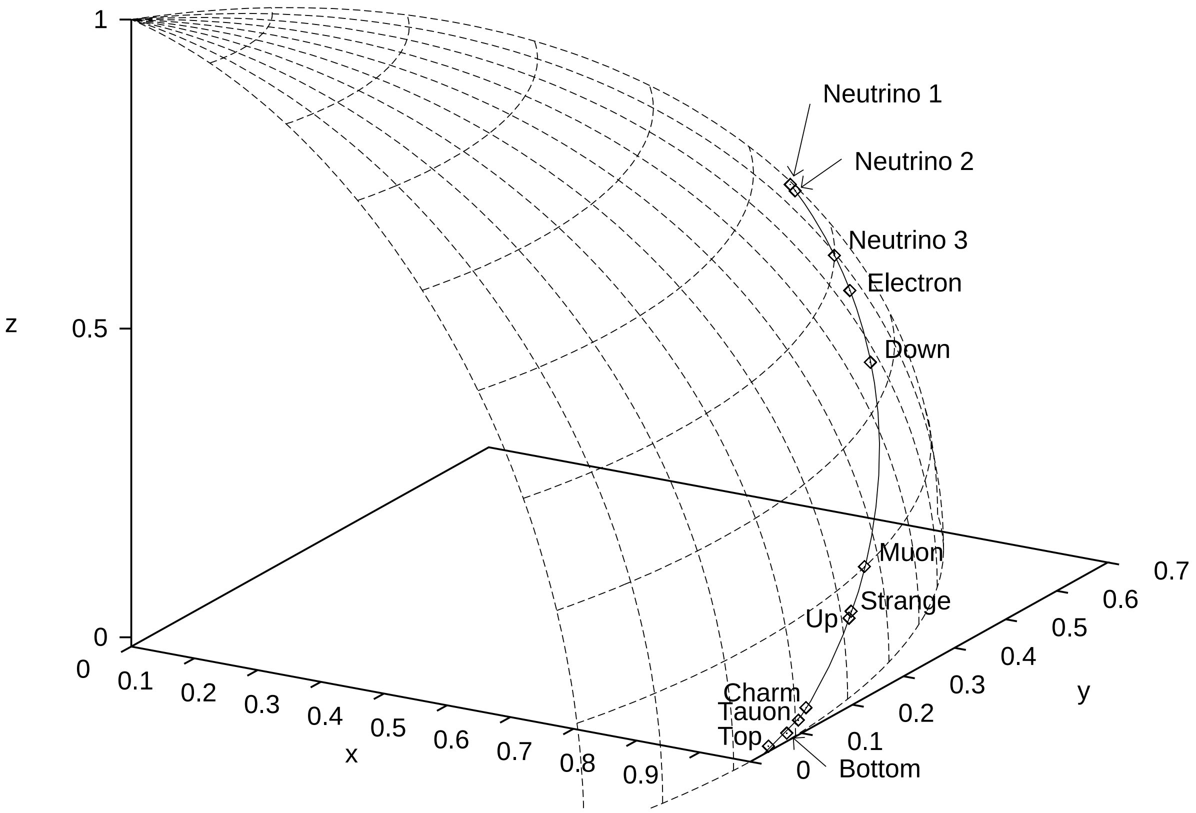
- does not change factorized form
- $\frac{d}{d(\ln \mu^2)} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{3}{64\pi^2} \rho^2 \begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix}$
- $\tilde{x}$ etc. simple known rational functions of x, y, z.

Mass matrix

- Normalized vector $v = (x, y, z)$ rotates with scale (w.l.o.g. $x \geq y \geq z$)
- Vector v lies on the unit sphere S^2
- Each type of fermion has its v: v_U, v_D, v_L, v_N
- ρ same for all types T
- $\rightsquigarrow v_U, v_D, v_L, v_N$ all on same RGE trajectory on S^2

Most of the results come from study of this rotating unit vector.

Trajectory of v



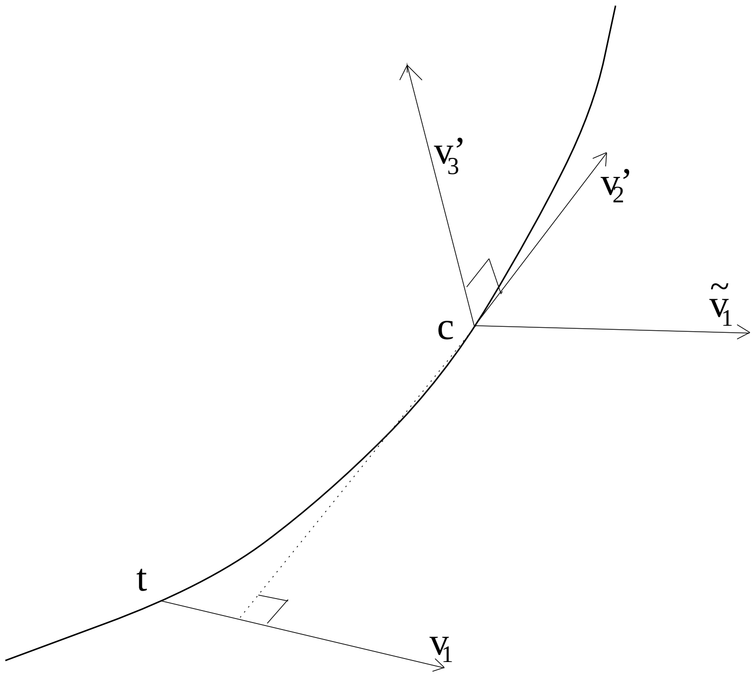
Fermion masses and states

- m has only one nonzero eigenvalue
- eigenvalues and eigenvectors depend on scale μ
- RGE has 2 fixed points:
  - $v = (1, 0, 0)$ at high energy
  - $v = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$ at low energy

A PROPOSAL: a working criterion

1. run m to scale where $\mu = m_1(\mu)$
2. corresponding e-vector is state vector v_1
3. do same with 2×2 remaining submatrix

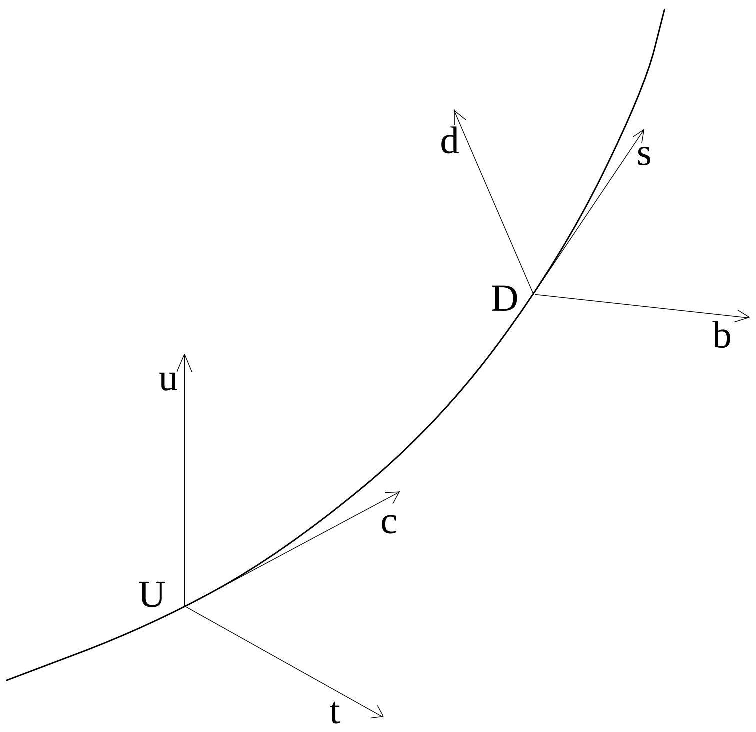
Fermion masses and states



- lepton state vectors always orthogonal
- mixing matrix always unitary
- mass hierarchy automatic

Fermion mixing

Gauge eigenstates $\neq$ mass eigenstates



Mixing matrix = direction cosines of the 2 triads

The Quark CKM Matrix

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Experimental

$$\begin{pmatrix} 0.9742-0.9757 & 0.219-0.226 & 0.002-0.005 \\ 0.219-0.225 & 0.9734-0.9749 & 0.037-0.043 \\ 0.004-0.014 & 0.035-0.043 & 0.9990-0.9993 \end{pmatrix}$$

Theoretical

$$\begin{pmatrix} 0.9752 & 0.2215 & 0.0048 \\ 0.2211 & 0.9744 & 0.0401 \\ 0.0136 & 0.0381 & 0.9992 \end{pmatrix}$$

The Leptonic MNS Matrix

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Experimental

$$\begin{pmatrix} * & 0.4 - 0.7 & 0.0 - 0.15 \\ * & * & 0.56 - 0.83 \\ * & * & * \end{pmatrix}$$

Theoretical

$$\begin{pmatrix} 0.97 & 0.24 & 0.07 \\ 0.22 & 0.71 & 0.66 \\ 0.11 & 0.66 & 0.74 \end{pmatrix}$$

Mixing pattern from classical differential geometry

Serret–Frenet–Darboux formulae for two neighbouring triads $\{N, T, B\}$ at Δs apart

$$\begin{pmatrix} 1 & -\kappa_g \Delta s & -\tau_g \Delta s \\ \kappa_g \Delta s & 1 & \kappa_n \Delta s \\ \tau_g \Delta s & -\kappa_n \Delta s & 1 \end{pmatrix}$$

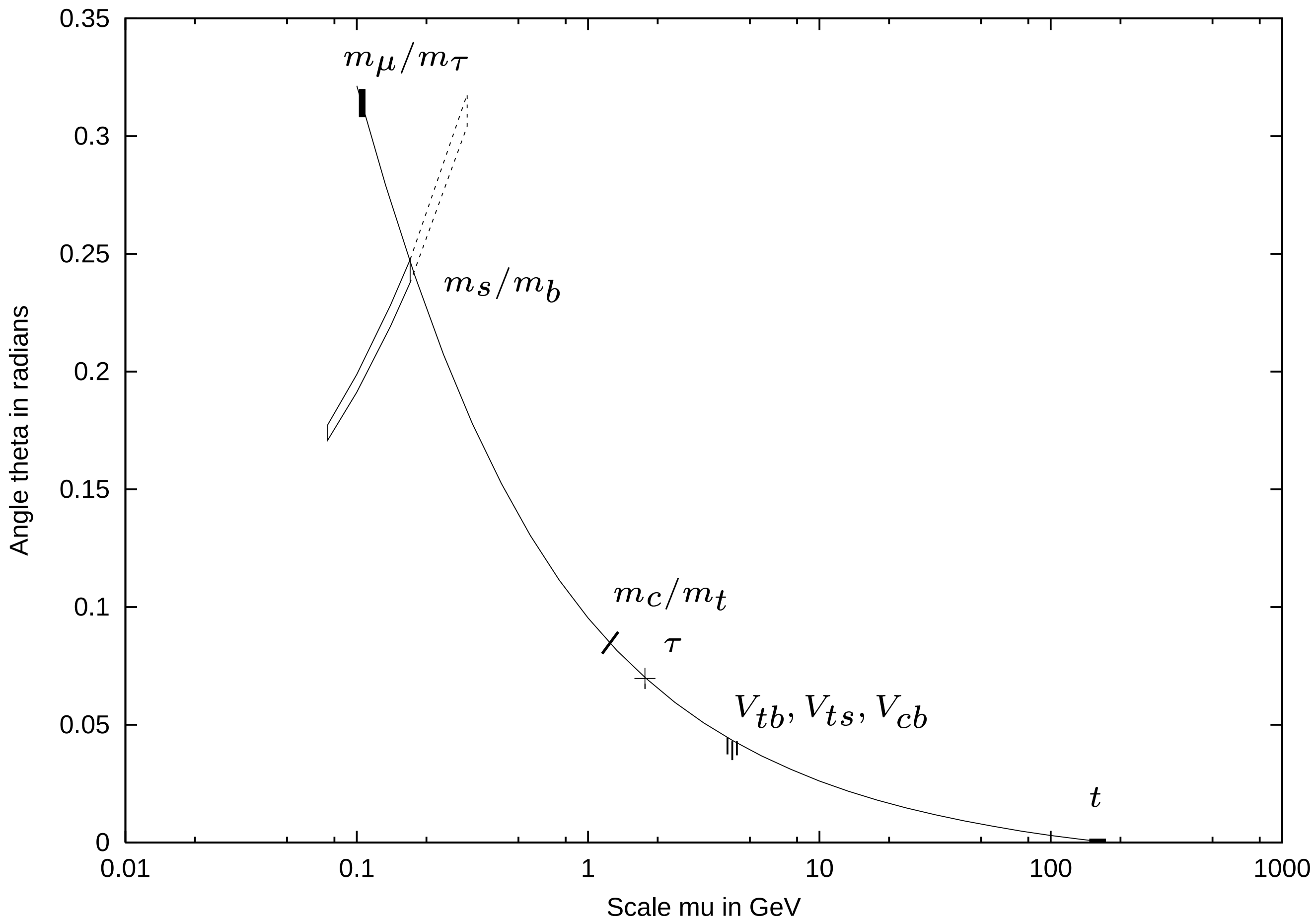
$$\overset{\text{sphere}}{\rightsquigarrow} \quad \begin{pmatrix} 1 & -\kappa_g \Delta s & 0 \\ \kappa_g \Delta s & 1 & \Delta s \\ 0 & -\Delta s & 1 \end{pmatrix}$$

N=normal to surface
T=tangent to curve
κ_g=geodesic curvature
κ_n=normal curvature(=1)
τ_g=geodesic torsion(=0)

Neutrinos

- In standard SM only ν_L so massless
- Can introduce ν_R as for other fermions
- ν_R can have large Majorana mass
- See-saw mechanism produces small physical ν mass
- $\begin{pmatrix} 0 & M \\ M & B \end{pmatrix} \rightsquigarrow$ one small eigenvalue
- ν_τ, ν_μ, ν_e not mass eigenstates
- $\rightsquigarrow$ ν oscillations
- DSM can naturally incorporate this feature

Near high energy fixed point



- Lowest generation masses u, d, e sensibly hierarchical but numerically inaccurate

- Their state vectors good because fixed at second generation positions

Near low energy fixed point

- Should not extrapolate 1-loop calculation over whole energy range

- Scale of $m_{\nu_3} \sim 0.05$ eV very near low energy fixed point

- Can approximate state vector of ν_3 by fixed point $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$: confirmed by calculation

- Get near maximal 'atmospheric' angle $U_{\mu 3}$ and small 'Chooz' angle U_{e3}: again confirmed by calculation

- 'Solar' angle U_{e2} not so good, as depending on how trajectory approaches fixed point (i.e. tangent)

Other consequences of DSM

Three broad areas of application

- Exchange of dual colour gauge bosons $\rightsquigarrow$ flavour-changing neutral current effects $\rightsquigarrow$ a lower bound for mass $\sim$ 500 TeV
  - rare hadron decays e.g. $K_L \to e^{\pm}\mu^{\mp}$
  - mass differences e.g. $K_L - K_S$
  - coherent muon-electron conversion on nuclei e.g. $\mu^- + Ti \to e^- + Ti$
  - muonium conversion e.g. $\mu^+ e^- \to e^+ \mu^-$
  - neutrinoless double beta decay e.g. $^{76}Ge \to {}^{76}Se + 2e^-$

- Rotating mass matrix $\rightsquigarrow$ lepton flavour violation or 'transmutation'
  - decays e.g. $\Upsilon \to \mu^{\pm}\tau^{\mp}$
  - photo-transmutation e.g. $\gamma e^- \to \gamma \tau^-$
  - transmutational Bhabha e.g. $e^+ e^- \to e^+ \mu^-$

- ultra high energy neutrinos

Air Showers

Cosmic rays with energy $> 10^{20}$ eV (beyond the Greisen–Zatsepin–Kuz'min bound) pose a problem in astrophysics:

- about 12 events over last 30 years
- each event produces about 10^{11} charged particles
- if protons will lose energy quickly by $p+\gamma_{2.7} \to \Delta+\pi$
- GZK: if proton then nearer than 50 Mpc away
- no obvious proton source that near
- some possible pairs or triples, would have been deflected if proton
- weakly interacting neutrino not enough cross-section with air neuclei

Air Showers

DSM offers a possible solution—at energy above dual colour gluon mass neutrinos will have become strongly interacting

GZK: lower bound of ~ 500 TeV (cf. FCNC)

- ν can escape strong em field around any source, e.g. AGN
- ν can survive long journey through microwave background
- Near hadronic cross-section with air nuclei
- Pairs (or triplets) not deflected by inter-galactic em field
- Highest energy event at 3×10^{20} eV with no abundant lower energy events in same direction: ν strongly only at high energy

To be done . . .

- Understand dual transform
- Further study Higgs field as frame vectors
- Geometric origin to Yukawa terms?
- Better picture of middle-energy range
- Further understanding of neutrinos
- and more . . .