

A brief introduction to Ramsey Theory

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If you are in a room with six people, can you find amongst those six people, three mutual strangers or three mutual acquaintances? It turns out that you can. However, if there are only five people in the room, then you may not always be able to do so.

The above problem can be represented by a graph. Mathematically, a graph is a collection of points (vertices) with lines (edges) joining them. If we have a graph with six vertices each representing a person and the edges joining any two vertices are coloured red or blue depending on the relation between the two people, say red for strangers and blue for acquaintances. Then the task of finding the three mutual strangers or acquaintances is equivalent to finding a red or a blue triangle within this graph.

Suppose we have a complete graph of order n , that is a graph with n vertices with an edge joining any two vertices. And suppose further the edges are coloured either blue or red. Then the *Ramsey Number* $R(p, q)$ is the minimum order required to guarantee we can find p vertices within the graph where all the edges are blue between them or q vertices where all the edges between them are red. Given these definitions, the fact that we need at least six people to guarantee three mutual strangers or acquaintances is telling us that $R(3, 3) = 6$.

We can think of the determination of Ramsey Numbers as a question of: *In how much global disorder can we find some local order?* The study of Ramsey Theory is a branch of discrete mathematics. It has applications in computer sciences and other branches of pure mathematics.

To this date, asymptotic (approximate) results have been obtained for the numbers of the form $R(3, k)$. The evaluation of the more general Ramsey Numbers $R(p, q)$, especially when p and q are large, remains a challenge to the mathematics community.