MACAULAY2 EXERCISES

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Remember that `viewHelp` (or `help` if you do not have your browser integrated properly) is a very useful command!

Warm-ups.

(1) Write a function isEven that takes as input an integer and returns 1 if it is even, and 0 if it is odd. A useful command is `%` :

```
i1 : 5 % 3

o2 = 2
```

Use `==` to test equality.

(2) The ideal I showed during the presentation is the ideal of the image of the 3-uple Veronese embedding of $\mathbb{P}^1$ into $\mathbb{P}^3$. We will now compute this in another way.
  (a) Create a polynomial ring R in two variables.
  (b) Create a list of the degree 3 monomials in your ring. The command `basis(3,R)` will help, as will the commands `entries` and `flatten`.
  (c) Create a polynomial ring in four variables S.
  (d) Create a homomorphism from S to R that sends the ith generator of S to the ith element of your list. The syntax for this is `map(R,S,yourList)`.
  (e) Compute the kernel of this map. The relevant command is `kernel`. Compare this with the ideal in presentation.

(3) (If you are familiar with Gröbner bases) Compute a Gröbner basis for the ideal $\langle x^3y^2 - 4x^2y^3 + 5y^5, x^6 - 7xy^5\rangle \subseteq \mathbb{Q}[x, y, z]$. Is $xy^9 \in I$? (Use `%` again).

Singularities of the space of non-invertible matrices.

(1) Create a 3×3 matrix of variables. You may want to name your variables

```
x_{1,2}
```

If you already have a variable named `x`, then you will need to clear it first using `x = symbol x`.

(2) Compute the determinant of your matrix. Remember that when the determinant of A is 0, the matrix A is not invertible
(3) Treating the determinant as a polynomial in the matrix entries, compute partial derivatives (the command `diff` will be useful here)
(4) Compute the ideal of partial derivatives (here we are computing the equations given by $\nabla f = 0$)
(5) Check that this is ideal is the same as the ideal of 2×2 minors of A (try `minors(2,A)`)
(6) Can you generalise this to $d = 4$ from $d = 3$? To check your generalization, the command `jacobian ideal f` will be useful to generate the ideal of partials of a polynomial f succinctly.

The Grassmannian. The Grassmannian $\mathrm{Gr}(d, n)$ is a variety that parameterises all d-dimensional subspaces of an n-dimensional vector space.

(1) Create a 2×4 matrix with generic entries (e.g., x_{ij}).
(2) Compute the six 2×2 minors of your matrix. The command `gens minors(2,A)` will produce a matrix with these entries.
(3) Compute a homomorphism from a polynomial ring in six variables to your ring that takes the ith generator to the ith minor on your list.
(4) Take the kernel of your homomorphism. This is the ideal of the Grassmannian $\mathrm{Gr}(2, 4)$. If you already knew what this variety was, compute the dimension to check that this is correct.
(5) Now write a function that takes as input your choice of $d < n$ to replace 2 and 4.
(6) This command actually already exists in Macaulay2! Look at the help for `Grassmannian`. (This command uses the projective convention for the Grassmannian, so to see our example you should type `Grassmannian(1,3)`). How can you use this to test that your function is correct?

Where next?

(1) Read the tutorials on the M2 webpage, starting with the first one.
(2) Have a look at the list of packages written by other people that are distributed with `Macaulay2`. Find one that is close to your research interests, and look at the help. Install it (use `needsPackage` or `installPackage`), and try some commands.