POLYHEDRAL GEOMETRY AND `POLYMAKE` EXERCISES

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(1) *Warm-up*
   (a) Initiate the 4-cube P.
   (b) How lattice points does P have?
   (c) How many interior lattice points does P have?
   (d) Compute the Ehrhart Polynomial of P.

(2) *Reflexive polygons*
   (a) Initiate a polytope $P = \mathrm{Conv}((1,0),(0,1),(-1,-1)) \subseteq \mathbb{R}^2$. See Figure 1.
   (b) Is P reflexive?
   (c) Compute its dual. Call it $Q = P^\vee$. Print the vertices (I recommend drawing Q on some graphed paper).
   (d) Show that

$$|\partial P \cap \mathbb{Z}^2| + |\partial Q \cap \mathbb{Z}^2| = 12.,$$

   that is, that the number of lattice points on the boundaries of P and Q is 12.
   (e) Display the properties which polymake has computed about P.

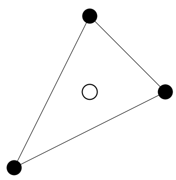

FIGURE 1. The polytope P.

(3) *Simplices*
   (a) Initiate the polytope $P = \mathrm{Conv}\left((4,0,0),(0,4,0),(0,0,2)\right)$. What is its dimension?
   (b) Is P a smooth polytope?
   (c) Define a matrix $M = (v_1 \,|\, v_2 \,|\, v_3)^t$. What is the null space of this matrix?
   (d) Is P a simplex?

(4) *Cones*
   (a) Initiate the cone $C = \mathrm{Cone}((4,-1),(1,0))$.

(b) Compute a Hilbert basis of C.

(c) Is C smooth?

(d) Check your answer using polymake.

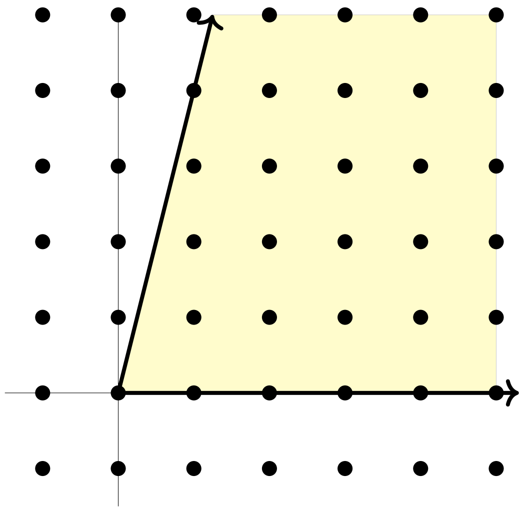

FIGURE 2. The cone C.