Knots exercises

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(1) Initialize the knot K corresponding to the braid with index 2 and braid notation: $[1, 1, 1]$.
- Compute the Gauss code of K.
- Compute the determinant, signature, Alexander and Jones polynomials of K.
- Plot the knot $K2$ obtained as the connected sum of K with itsels, and compute all the previous invariants.

(2) Generate all the knots in the database having crossing number less or equal than 6. Among those knots, find those having determinant $= 5$

(3) Which one of the following knots (given as oriented gauss codes) has Alexander polynomial $= 1$ (same as the unknot)? Which one is trivial? (Hint: the Alexander polynomial does not detect the unknot while the Jones polynomial conjecturally does).
- $[-1, 2, -4, 5, 7, -10, 11, -7, 8, -9, 10, -11, -3, 4, -6, -8, 9, 1, -2, 3, -5, 6]$, $[-1, -1, 1, -1, 1, -1, 1, -1, -1, 1, 1]$
- $[1, -4, 3, -1, 10, -9, 6, -7, 8, 5, 4, -3, 2, -6, 7, -8, 9, -10, -5, -2]$, $[1, -1, 1, 1, 1, -1, -1, -1, -1, -1]$
- $[1, -2, 3, -4, 5, -6, 7, -1, 2, -3, 4, -5, 6, -7]$,$[1, 1, 1, 1, 1, 1, 1]$

(4) Check in the list of point 2) that the absolute value of the determinant coincides with the Jones polynomial evalueted in -1 and the Alexander polynomial evalueted in -1. Check on some example that the signature is additive under connected sum, and changes sign when taking the mirror.

(5) Generate a random braid of braid index 5 with 10 crossings, plot it and compute the number of its components.