Abram Gannibal Project Visitor Programme 2021-22
List of potential projects

Diane Maclagan (University of Warwick), Balázs Szendrői (University of Oxford) and Michael Wemyss (University of Glasgow): Computer algebra and applications

All these projects will start with an introduction to the basics of computer algebra, looking at freely available software such as Singular [8] and Macaulay2 [14], or the propriatory software Magma [12], as appropriate (for the basics of algorithms used in computer algebra, see [3]). Following that, some of the following topics will be explored in detail.

a. Coding in graphs, and implementing various simple algorithms

b. Grobner basis problems in either commutative or noncommutative rings; e.g. implementing the Ufranoski graph from a given Grobner basis

c. Problems in coding finite dimensional algebras, and implementing simple algorithms to work out their basic properties

d. Coding properties of the singular locus, Jacobi ideals, resolutions of singularities, classification of curve singularities

e. Representation theory and algebraic geometry: equations of homogeneous varieties such as Hilbert schemes

f. Tropical geometry [15]

Some of the projects might involve collaboration on a new Macaulay2 package.

Hamid Ahmadinezhad (Loughborough University): Theory of bonds in kinematics, and linkages with helical joints

A linkage consists of a number of rigid bodies (robot arms) connected via joints that allow rotation or other movements, and they are used to describe robot configurations. A linkage is called closed when the arms glue together to make a loop. They are useful in practice as a simple force generates movements for the linkage, without any need for further control. Closed linkages can be modelled using dual quaternion algebras, and their mobility can be studied using algebra-geometric methods. In this project, we will study the mathematical foundations behind closed linkages and explore the theory of bonds, which is used to predict mobility of a given linkage. This will be coupled with many practical examples and computational tools. A strong emphasis will be put on closed linkages with helical joints, which present several open questions. A good starting point is the book [13]; a recent paper is [1].

Agnese Barbensi (University of Oxford): Topological methods for open knots

This project will explore the concept of open knots, and how to efficiently characterise their entanglement. Since any open curve in S^3 or $\mathbb{R}^3$ can be deformed into a straight, crossingless segment through ambient isotopies, there is no immediate way to assign a specific knot type to it, or even to decide whether it is knotted or not. A classical approach to identify the topological type of open curves in space consists in considering probabilistic closures of the chain [16] to create a distribution of well defined mathematical knots. More recently, the introduction of mathematical objects called knotoids [17] inspired a new way to tackle this problem. In this approach, the topology of an open curve is described by analysing its planar projections as knotoids [9]. In this project, we will study and compare these different approaches, with a focus on applications to biology [5] and on how to use the available software to analyse entangled polygonal curves arising in nature [7, 4].

Ambrus Pál (Imperial College London): p-adic analysis and applications

This project will explore the theory of p-adic numbers and p-adic analysis, and their links to arithmetic geometry. In particular, we will look at the proof of the rationality of the zeta functions of algebraic varieties over finite fields following work of Dwork. This method can be refined for special subvarities to give explicit formulas or congruences, and there have been a couple of papers on this topic recently, which will give us ideas for further research. The problem of effective point counting on algebraic varieties over finite fields also has potentially far reaching applications in cryptography, especially if elliptic curve crypto-systems become compromised, and we will also look at this aspect of the theory. A short introduction to p-adic numbers is [2]; a more detailed treatment and a review of work of Dwork can be found in the classical textbook [10].

Gregory Sankaran (University of Bath): Cremona transformations and applications

A *Cremona transformation* is a birational automorphism $\varphi\colon \mathbb{P}^n \dashrightarrow \mathbb{P}^n$ of n-dimensional projective space $\mathbb{P}^n$: that is, a change of coordinates effected by higher-degree polynomials rather than linear ones. Perhaps the simplest example is $(x : y : z) \mapsto (yz : zx : xy)$, but there are very many of them: the Cremona transformations of $\mathbb{P}^n$ form a huge (infinite-dimensional) group. Nevertheless, a lot is known about particular kinds of Cremona transformation. The first purpose of this project will be to familiarise ourselves with some of this theory, in such a way as to be able to use it in practice. A rather comprehensive introduction, written in fairly modern language, may be found in [6]. Because changing coordinates is a natural thing to do, Cremona transformations can arise in contexts quite remote from algebraic geometry, including engineering contexts. The second purpose of the project will be to examine some of these.

References

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