Knots and SageMath

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AGP online seminar and workshop

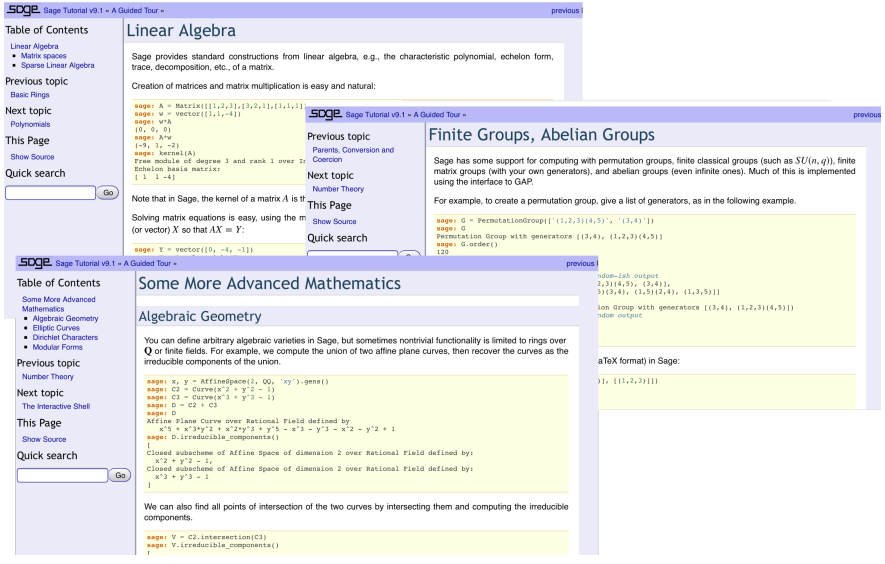
UNIVERSITY OF OXFORD
Mathematical Institute

Oxford Mathematics

SageMath: an open source mathematics software

www.sagemath.org/

Basic and Advanced Mathematics with SageMath!



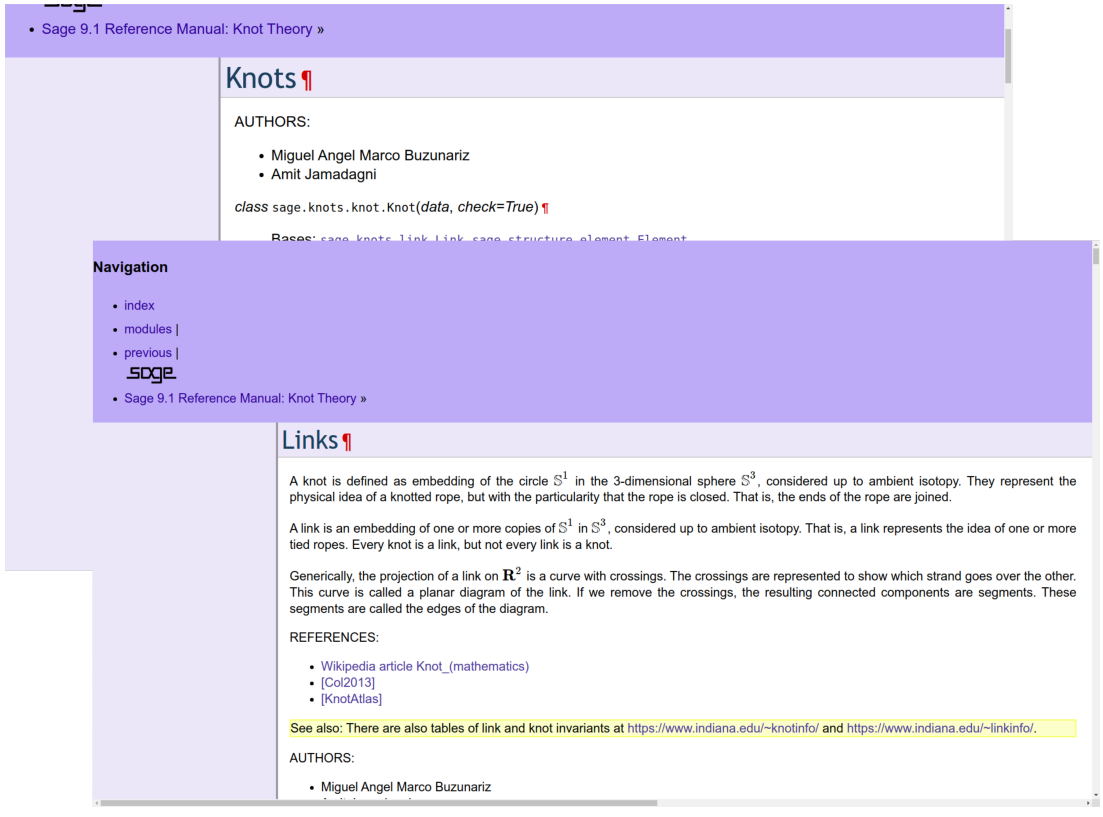
Download it at:

- sagemath.org/download.html

or use it **online** at:

- sagecell.sagemath.org
- cocalc.com

Knots & Links classes on SageMath



- Sage 9.1 Reference Manual: Knot Theory »

Knots ¶

AUTHORS:

- Miguel Angel Marco Buzunariz
- Amit Jamadagni

class `sage.knots.knot.Knot`(*data*, *check=True*) ¶

Bases: sage.knots.link.Link, sage.structure.element.Element

Navigation

- index
- modules |
- previous |

- Sage 9.1 Reference Manual: Knot Theory »

Links ¶

A knot is defined as embedding of the circle $\mathbb{S}^1$ in the 3-dimensional sphere $\mathbb{S}^3$, considered up to ambient isotopy. They represent the physical idea of a knotted rope, but with the particularity that the rope is closed. That is, the ends of the rope are joined.

A link is an embedding of one or more copies of $\mathbb{S}^1$ in $\mathbb{S}^3$, considered up to ambient isotopy. That is, a link represents the idea of one or more tied ropes. Every knot is a link, but not every link is a knot.

Generically, the projection of a link on $\mathbf{R}^2$ is a curve with crossings. The crossings are represented to show which strand goes over the other. This curve is called a planar diagram of the link. If we remove the crossings, the resulting connected components are segments. These segments are called the edges of the diagram.

REFERENCES:

- Wikipedia article Knot_(mathematics)
- [Col2013]
- [KnotAtlas]

See also: There are also tables of link and knot invariants at https://www.indiana.edu/~knotinfo/ and https://www.indiana.edu/~linkinfo/.

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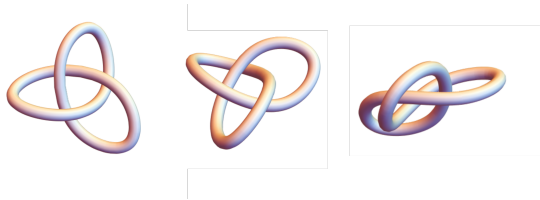
https://doc.sagemath.org/html/en/reference/knots/sage/knots/link.html

Knots and Links

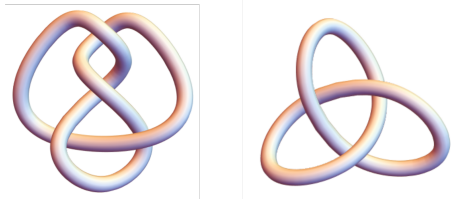
Knots: embeddings of $S^1 \hookrightarrow S^3$ up to **ambient isotopy**.



Often studied through their **diagrams**.

Links are disjoint unions of knots.

Question: How to tell whether or not two knots are different?



Invariant :

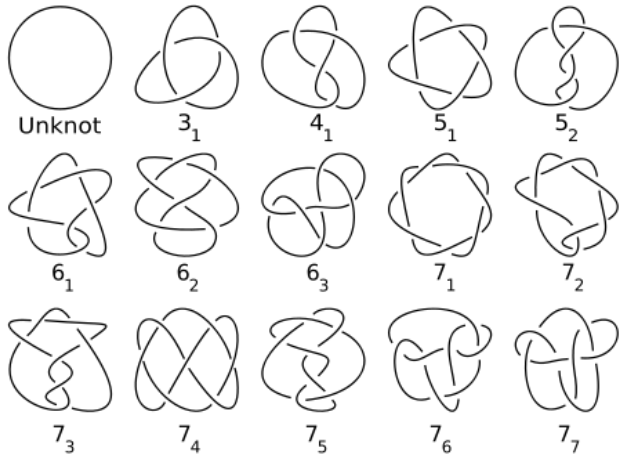
"Knots" $\longrightarrow$ "something algebraic"

such that if two knots have different images, then they are distinct.

Distinguish Knots

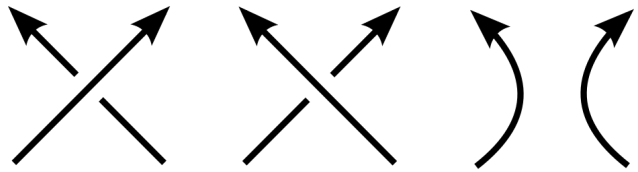
Knot theory's main goal: distinguish and classify knots.

- **Crossing number** of K: minimum number of crossings in a diagram of K
- **Knot polynomials:** computed using **skein relations**



Distinguish Knots

Knot theory's main goal: distinguish and classify knots.



L_+ L_- L_0

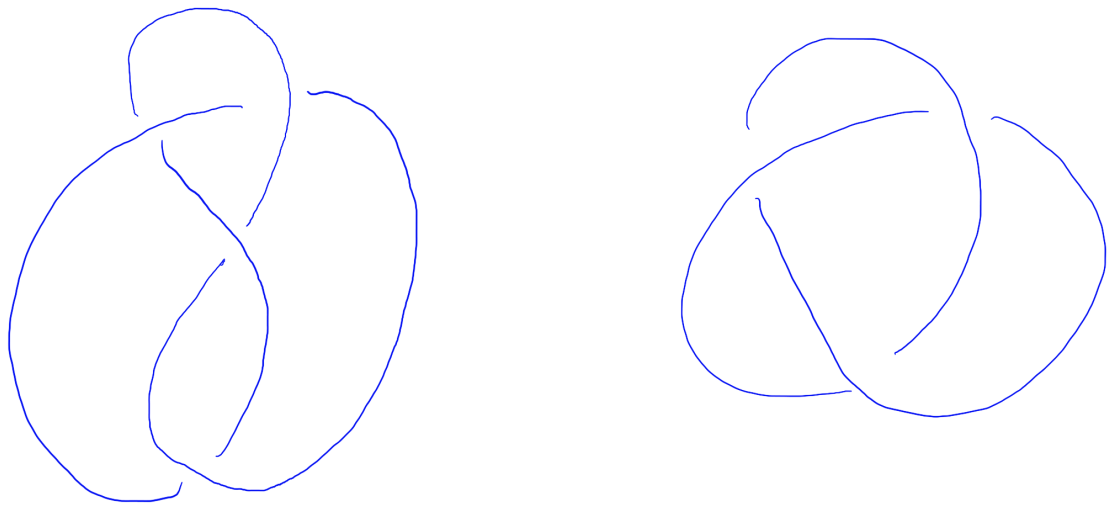
- **Crossing number** of K: minimum number of crossings in a diagram of K
- **Knot polynomials:** computed using **skein relations**:

Alexander Polynomial:

- **Normalisation** on the trivial knot $\Delta_{\bigcirc}(t) = 1$
- $\Delta_{L_+}(t) - \Delta_{L_-}(t) = \left(t^{\frac{1}{2}} - t^{-\frac{1}{2}}\right) \Delta_{L_0}(t)$

Operation on Knots.

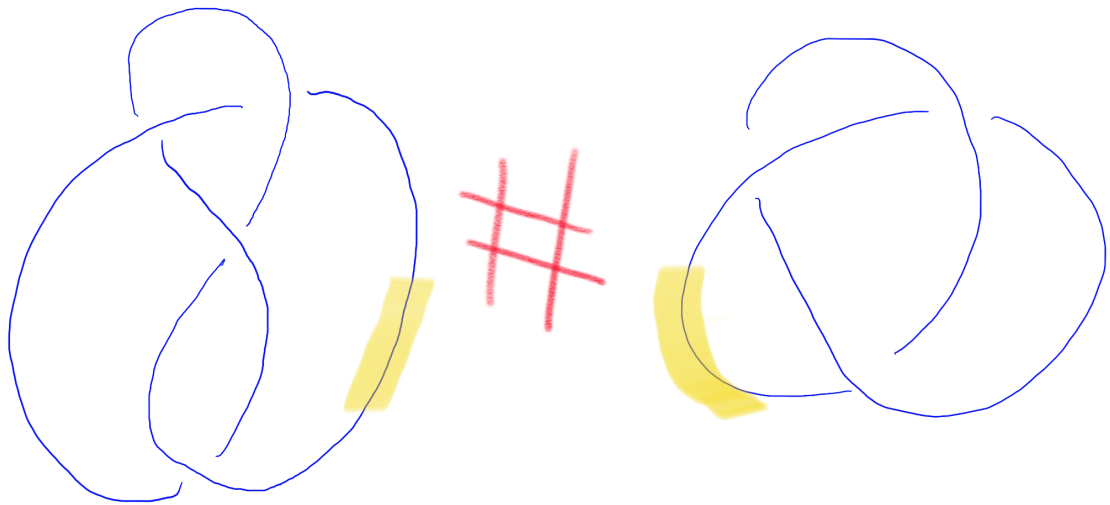
Connected **sum:**



Take a disjoint diagram of K_1 and K_2.

Operation on Knots.

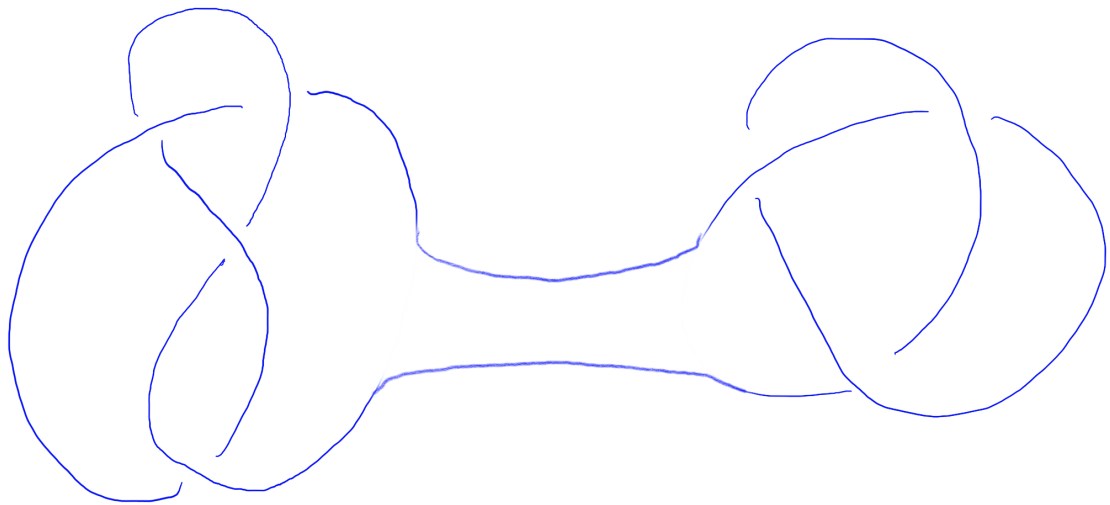
Connected **sum:**



Select an arc from each diagram.

Operation on Knots.

Connected **sum:**

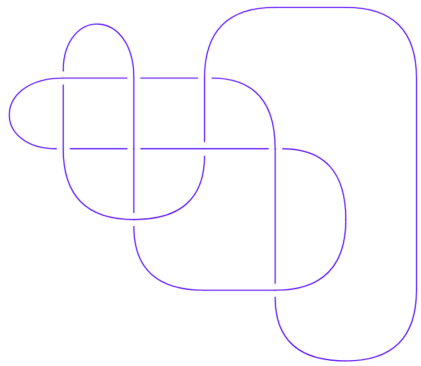


Remove the arcs and join them as shown.

More on knot invariants

- **Integer valued** invariants: crossing number, determinant, signature...
- **Knot polynomials:** Alexander, Jones, Kauffman, HOMFLY...
- **Knot homologies:** Khovanov, Knot Floer...

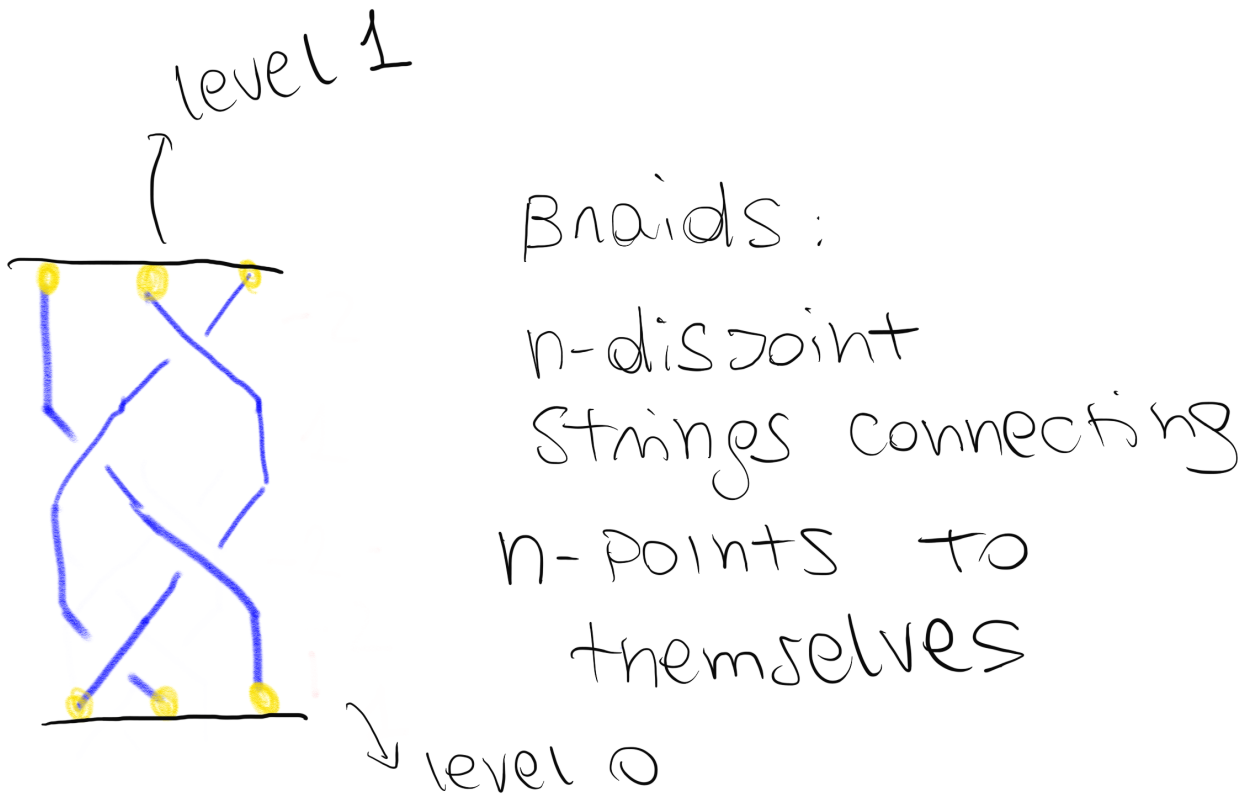
Most of these are computable on **SageMath**



```
sage: K.alexander_polynomial()
-2*t^-2 + 8*t^-1 - 11 + 8*t - 2*t^2
sage: K.jones_polynomial()
t^7 - 3*t^6 + 4*t^5 - 5*t^4 + 6*t^3 - 5*t^2 + 4*t + 1/t - 2
sage: K.determinant()
31
sage: K.signature()
-2
```

Knots initialization

As **braids** (Markov's Theorem)



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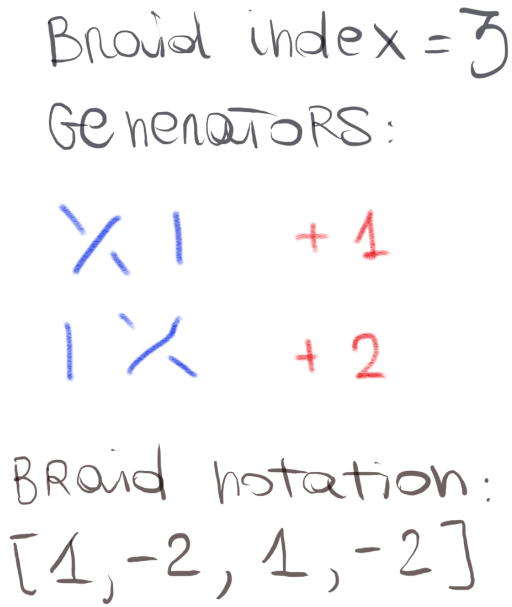

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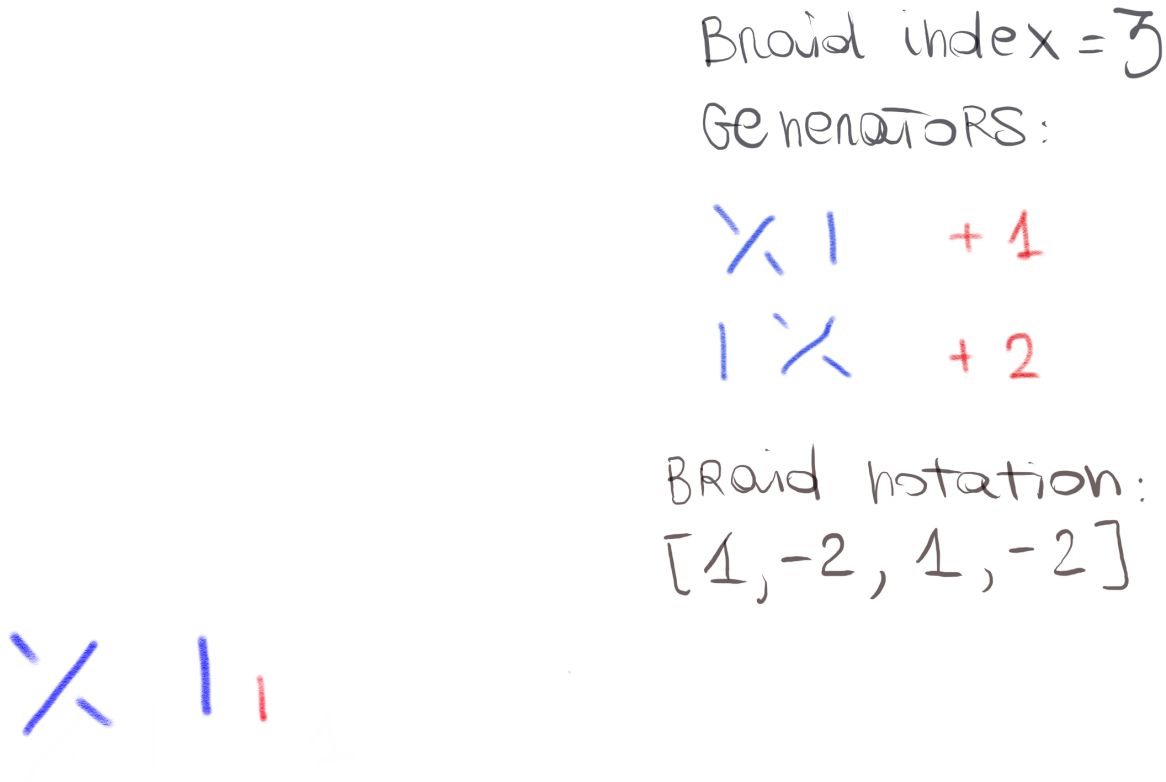

Knots initialization

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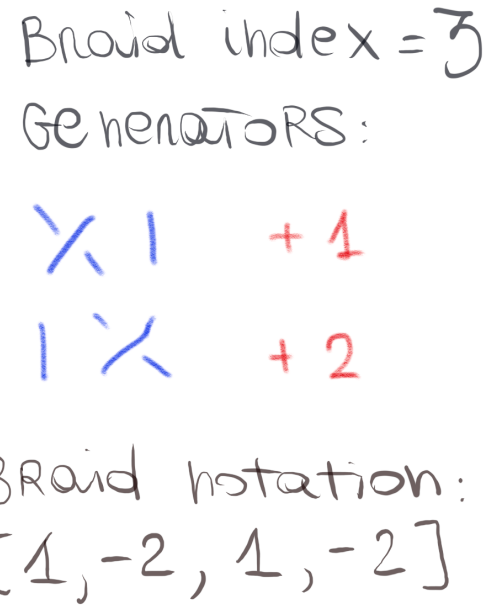

Knots initialization

As **braids** (Markov's Theorem)

Braid index = 3

Generators:

Braid notation:

[1, -2, 1, -2]

Knots initialization

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Knots initialization

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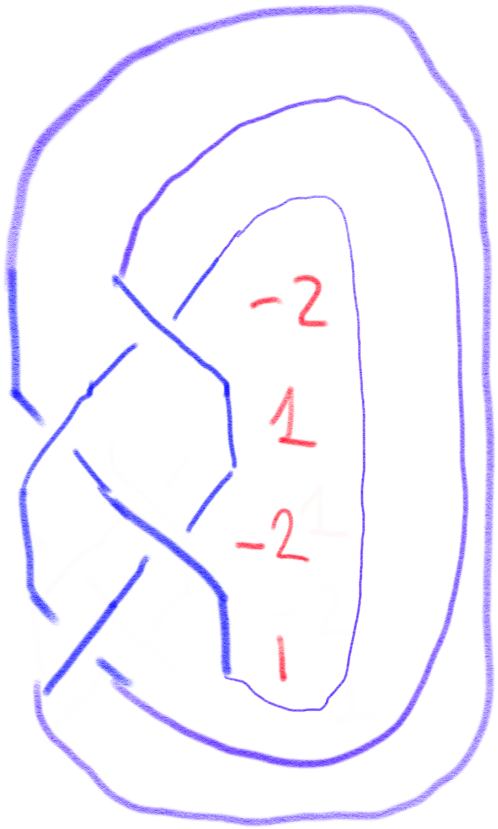


Braid index = 3

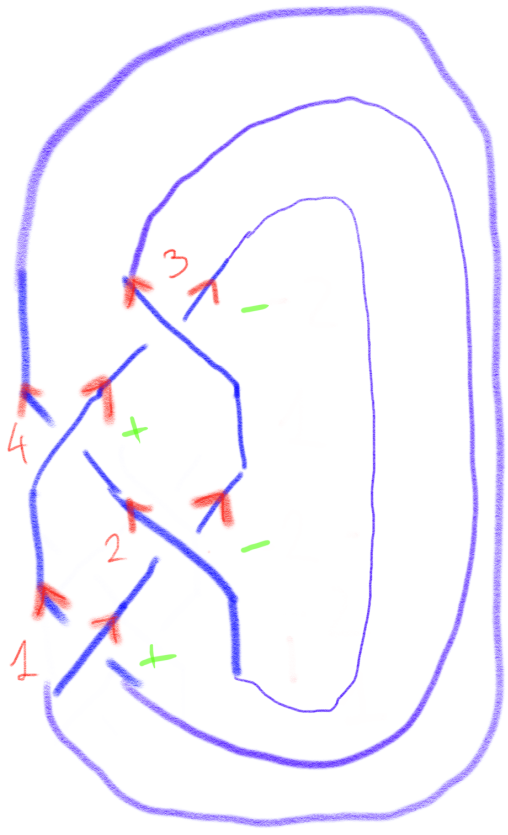
Generators:

Braid notation:

[1, -2, 1, -2]

Knots initialization

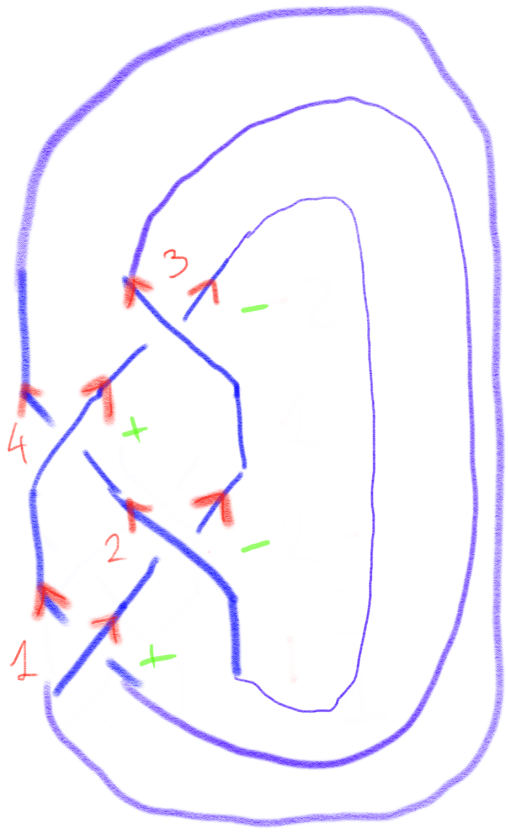
Using the **Oriented Gauss Code**



oriented
Gauss code:

Knots initialization

Using the **Oriented Gauss Code**



oriented
Gauss code:

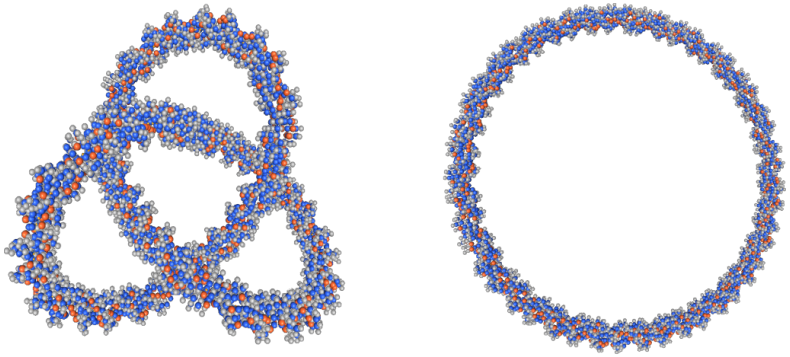
[1,-2,3,-1,4,-3,2,-4],

[1,-1,1,-1]

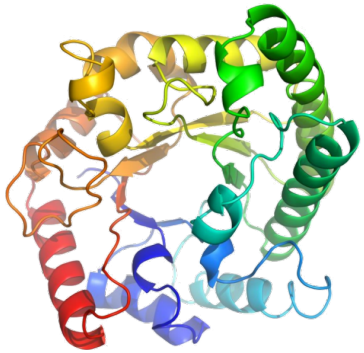
Let's see it on SageMath...

Real life applications:

- **Biology:** DNA topology, protein entanglement, clustering in Bio-informatics..



- **Physics:** vortices, solar flares...



And now a few exercises!

Thanks for the attention!!