Roughly, the Socle Lemma refers to the idea that, in a definable group of finite rank, a type with trivial stabiliser should be internal to rank 1 types. Hrushovski's original argument applied only to rigid abelian groups of finite rank. Under the assumption of the Canonical Base Property (CBP), Pillay and Ziegler [PZ03] gave a version for arbitrary finite rank groups.

In the paper under review, the authors prove a version of the Socle Lemma for rigid groups in simple theories, with no finite rank hypothesis. Roughly, a group is rigid if it has no definable families of subgroups. The Socle statement becomes that if $G$ is rigid and analysable in a family $\Sigma$ of partial types, then any type of an element of $G$ with bounded stabiliser is almost internal to $\Sigma$.

This is to be contrasted with the result of Hrushovski-Palacin-Pillay [HPP13] that a stable non-multidimensional theory in which all Galois groups are rigid has the CBP, and hence that a Socle Lemma holds.

There is no obvious relation between the assumption that the Galois groups are rigid and the assumption that $G$ in the statement of the Socle Lemma is rigid, nor is there an indication that rigidity of $G$ implies any version of CBP. Nonetheless, the authors of the present paper take the opportunity to use a result of Chatzidakis to mildly strengthen the HPP result, removing the assumption of non-multidimensionality.

The proof of the main Socle result involves an analysis of the properties of stabilisers and rigidity in groups (hyper)definable in simple theories. A knowledge of the basics of simplicity theory is assumed.

# References

[HPP13] Ehud Hrushovski, Daniel Palacín, and Anand Pillay. On the canonical base property. *Selecta Math. (N.S.)*, 19(4):865–877, 2013.

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