Recall that a complete first-order theory is VC-minimal if there are $\phi_i(x, \bar{y})$ such that in any model M, any formula with parameters is equivalent to a boolean combination of instances $\phi_i(x, \bar{b})$, and the boolean subalgebra of $\mathcal{P}(M)$ generated by any pair $(\phi_i(x, \bar{b}), \phi_j(x, \bar{c}))$ of instances is not free.

This notion was introduced by Adler as an abstraction of the "swiss cheese" decomposition admitted by algebraically closed valued fields, but as a tameness notion it has some apparent deficiencies: the reduct of a VC-minimal theory need not be VC-minimal, and VC-minimality has no known nice consequence for the structure of definable sets in more than one variable. This paper discusses three variants of VC-minimality, two of which are motivated by these problems.

The first of these: T is *convexly orderable* if some (any) model M can be equipped with a linear order such that any T-definable subset of M is a finite union of convex sets, with uniform bounds in definable families on the number of convex sets. (It seems that, in contrast with the situation with strong minimality and o-minimality, this explicit condition on uniformity of bounds can not be removed.)

Convex orderability is clearly preserved under taking reducts. The authors show that it is a strict weakening of VC-minimality, and that it implies that formulae in one variable have vc-density ≤ 1, and hence that it implies dp-minimality.

Secondly: *weak VC-minimality* relaxes the condition in the above definition of VC-minimality by restricting the assumption on pairs of instances to pairs of instances of a common formula, $(\phi_i(x, \bar{b}), \phi_i(x, \bar{c}))$. The authors show that this is a strict strengthening of NIP, and that in a weakly VC-minimal theory any $\phi(x, \bar{y})$ with the order property has the strict order property, and hence if the theory is unstable it interprets an infinite linear order; they deduce that the Keuker conjecture holds for weakly VC-minimal theories.

Lastly: *full VC-minimality* strengthens VC-minimality with a uniformity assumption, namely by requiring in the above definition that any formula without parameters is equivalent to a boolean combination of the $\phi_i(x, \bar{y})$. Full VC-minimality is a very strong condition; as the authors note, even ACF_0 does not satisfy it (while any strongly minimal theory is VC-minimal), although weak o-minimality does imply it.

The authors prove under this strong assumption a positive result about definable sets in many variables - namely "low VC-density", meaning that the VC-density of any $\phi(\bar{x}, \bar{y})$ is bounded by the length of $\bar{x}$. Whether this holds for VC-minimal theories remains an open question.

The paper concludes with some open questions.

The exposition throughout the paper is quite clear. Proofs are mostly presented in great detail.