~~Hello!~~

$$f(x) := \frac{1}{1+e^{-x}}$$

$$f'(x) = \frac{d}{dx}(1+e^{-x})^{-1}$$

$$= -(-e^{-x})\cdot(1+e^{-x})^{-2}$$

$$= \frac{e^{-x}}{(1+e^{-x})^2}$$

$f'(x) = 0$

$\iff e^{-x} = 0$

never

$f'(x)$ has no zeroes

$f'(x) > 0$ for all x

so $\underline{f \text{ is increasing}}$

$$f''(x) = \frac{d}{dx}\left((e^{-x})(1+e^{-x})^{-2}\right)$$

$$= -e^{-x}(1+e^{-x})^{-2} + e^{-x}\cdot(-2)(-e^{-x})(1+e^{-x})^{-3}$$

$$= \frac{-e^{-x}}{(1+e^{-x})^2} + \frac{2e^{-x}e^{-x}}{(1+e^{-x})^3}$$

$$= \frac{-e^{-x}(1+e^{-x}) + 2e^{-x}e^{-x}}{(1+e^{-x})^3}$$

$$= \frac{e^{-x}e^{-x} - e^{-x}}{(1+e^{-x})^3}$$

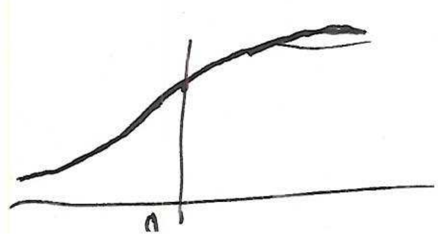
$$= \frac{e^{-x}(e^{-x}-1)}{(1+e^{-x})^3}$$

$f''(x) = 0 \iff e^{-x}(e^{-x}-1) = 0$

$\iff e^{-x} - 1 = 0$

$\iff x = 0$

x	$(-\infty, 0)$	0	$(0, \infty)$
$f''(x)$	$+$	0	$-$

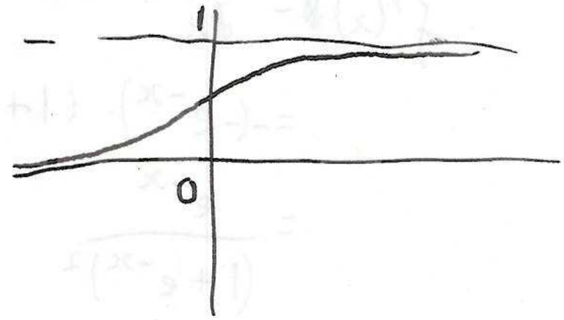


$$f(x) = \frac{1}{1+e^{-x}}$$

$$\lim_{x\to+\infty} f(x) = \frac{1}{1+\lim_{x\to+\infty} e^{-x}} = \frac{1}{1+0} = 1$$

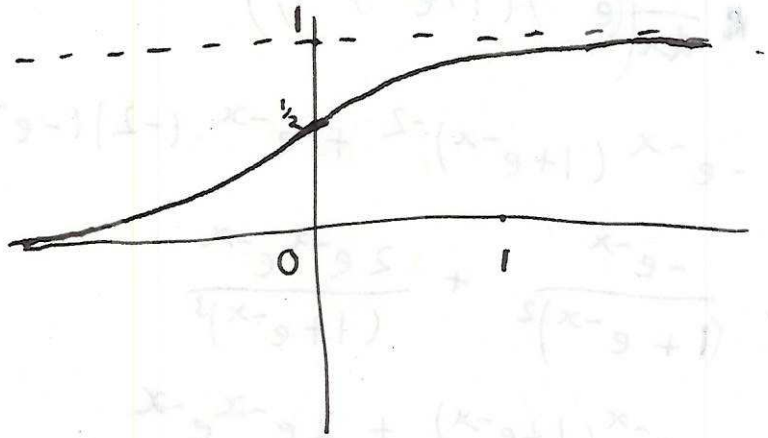
$$\frac{A}{1+Be^{-Cx}}$$

$$\lim_{x\to-\infty} f(x) = 0$$



$$f(0) = \frac{1}{1+1} = \frac{1}{2}$$

$$f'(0) = 1/4$$



f "logistic function"

"bounded exponential growth"

$$1-f(x) = 1 - \frac{1}{1+e^{-x}} = \frac{1+e^{-x}-1}{1+e^{-x}}$$

$$= \frac{e^{-x}}{1+e^{-x}}$$

$$= \frac{1}{1+e^{x}} = f(-x)$$

Antiderivatives

Def^n: An antiderivative of a function f is a function F such that

$$F'(x) = f(x)$$

for every x in the domain of f

e.g. if $s(t)$ is the position of an object on a line at time t
and $v(t)$ is its velocity
then s is an antiderivative of v

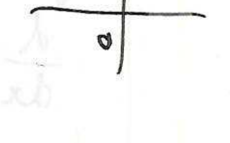
e.g. $\frac{d}{dx} x^2 = 2x$

so x^2 is an antiderivative of $2x$

$$\frac{d}{dx} \tfrac{1}{2} x^2 = \tfrac{1}{2} \frac{d}{dx} x^2 = \tfrac{1}{2} 2x = x$$

so $\frac{1}{2} x^2$ is an antiderivative of x

Similarly, $\frac{x^r}{r}$ is an antiderivative of x^{r-1}
for any $r \neq 0$ e.g. $\frac{x^3}{3}$ is an antid^{ve} of x^2



Note:
$$\frac{d}{dx}\left(\tfrac{1}{2}x^2 + 7\right) = \frac{d}{dx}\left(\tfrac{1}{2}x^2\right) + \frac{d}{dx} 7$$
$$= x + 0$$
$$= x$$

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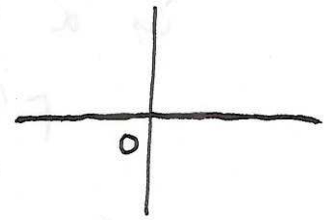
so $\frac{1}{2}x^2+7$ is also an antiderivative of x

similarly $\frac{1}{2}x^2+c$ for any c

What are the antiderivatives of 0?



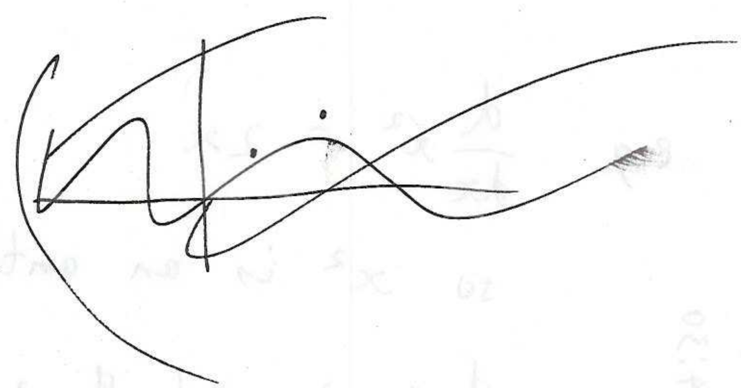
Any constant function is one

$(f(x)=1,\ f(x)=7,\ f(x)=e)$

Fact: $\frac{d}{dx}F(x)=0$ for all x

$\Leftrightarrow$ F is constant $(F(x)=c$ some $c)$

i.e. the antiderivatives of 0 are precisely the constant functions



Theorem: If f is a function and $\operatorname{dom} f$ is an interval and if F and G are antiderivatives of f then for some c, $F(x)=G(x)+c$

"antiderivatives are well-defined up to addition of a constant"

Proof (in the case $\operatorname{dom} f=\mathbb{R}$)

$\frac{d}{dx}F(x)=f(x)$ $\quad$ $\frac{d}{dx}G(x)=f(x)$

so $\frac{d}{dx}(F(x)-G(x))=\frac{d}{dx}F(x)-\frac{d}{dx}G(x)=f(x)-f(x)=0$

so $F(x)-G(x)=c$ some c, so $F(x)=G(x)+c$ $\square$