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We saw last time that e.g. the antiderivatives of x are $\frac{x^2}{2} + c$, c an arbitrary real

$$\left(\frac{d}{dx} \frac{x^2}{2} = \frac{1}{2} \frac{d}{dx} x^2 = \frac{1}{2} 2x = x \right)$$

We write

$$\int f(x)\, dx$$

to refer to an antiderivative of $f(x)$

we call $\int f(x)\, dx$ "the indefinite integral of $f(x)$ with respect to x"

e.g $\int x\, dx = \frac{x^2}{2} + c$

"constant of integration"

<u>Fact</u>: Every continuous function has an antiderivitive

e.g. $\frac{d}{dx} x^r = r x^{r-1}$ (any r)

so $\int x^r\, dx = \frac{x^{r+1}}{r+1} + c$ (any $r \neq -1$)

since $\frac{d}{dx} \frac{x^{r+1}}{r+1} = \frac{1}{r+1} \frac{d}{dx} x^{r+1} = \frac{1}{r+1}(r+1) x^r = x^r$

<u>Note</u>: if $r = -1$, $r+1 = 0$ so $\frac{x^{r+1}}{r+1} = \frac{1}{0}$ undefined

so what is $\int \frac{1}{x}\, dx$

Remark: "Integration is inverse to differentiation":

$$\frac{d}{dx}\int f(x)\,dx = f(x)$$

since $\int f(x)\,dx$ is an antiderivative of $f(x)$

e.g. $\int x^5\,dx = \frac{x^6}{6} + c$

$$\frac{d}{dx}\left(\frac{x^6}{6}+c\right) = \frac{6x^5}{6} + 0$$
$$= x^5$$

and $\int \frac{df}{dx}(x)\,dx = f(x) + c$

since $f(x)$ is an antiderivative of $\frac{df}{dx}$

e.g. $\frac{d}{dx}x^5 = 5x^4$

$$\int 5x^4\,dx = x^5 + c$$

What is this dx thing?

Recall: the "x" in "$\frac{d}{dx}$" indicates the variable with respect to which ~~fn~~ we are ~~differentiation~~ differentiating

e.g. $\frac{d}{dx}(x^2+xy) = 2x + y\frac{d}{dx}x = 2x + y$

$\frac{d}{dy}(x^2+xy) = 0 + x = x$ | $\frac{d}{dt}(x^2+xy) = 0$

Similarly for $\int \ldots dx$

e.g. $\int xy\,dx = \frac{x^2}{2}y + c$

$\int xy\,dy = x\frac{y^2}{2} + c$

$\int xy\,dt = xyt + c \qquad \left(\frac{d}{dt}xyt = xy\right)$

What is $\int \frac{1}{x}\,dx$?

$\int x^r\,dr = \frac{x^{r+1}}{r+1} + c$

$\frac{d}{dx}\ln x = \frac{1}{x}$

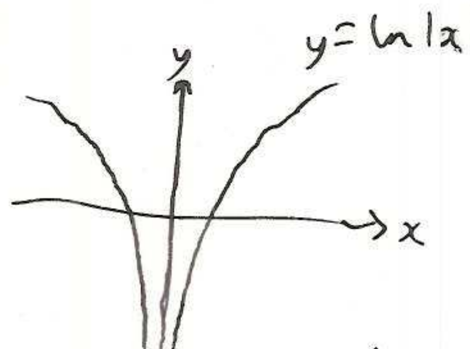
But: $\ln x$ is only defined for $x>0$

while $\frac{1}{x}$ is defined for any $x \neq 0$

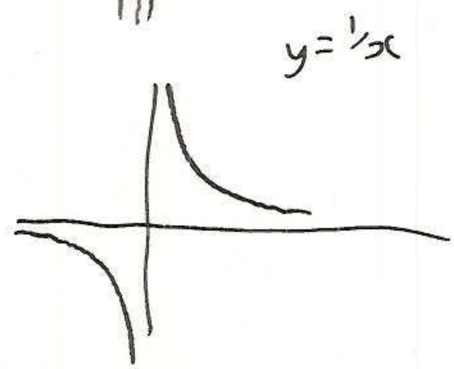
cunning trick: $\frac{d}{dx}\ln(-x) = -\frac{1}{-x} = \frac{1}{x}$

$\ln(-x)$ defined $x<0$

so $\ln|x| = \begin{cases} \ln x & x>0 \\ \ln(-x) & x<0 \\ \text{undefined} & x=0 \end{cases}$



$\frac{d}{dx}\ln|x| = \frac{1}{x}$ all $x \neq 0$



so $\int \frac{1}{x}\,dx = \ln|x| + c$

$$\int e^x\,dx = e^x + c$$

$$\int e^{2x}\,dx = \frac{1}{2}e^{2x} + c$$

$$\left(\begin{aligned}\frac{d}{dx}\frac{1}{2}e^{2x} &= \frac{1}{2}\frac{d}{dx}e^{2x}\\ &= \frac{1}{2}2e^{2x} \quad \text{(chain rule)}\\ &= e^{2x}\end{aligned}\right)$$