Midterms;

2nd 1st March } Thursday
3rd 22nd March

conflict ⇒ mail lozinski@math.mcmaster.ca

specify schedule for the day and the following morning

asap!

Lec 17;

Recall; Riemann sum definition of integral:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \Delta x \sum_{i=1}^{n} f(x_i) \qquad \Delta x = \frac{b-a}{n}$$

for each n; x_i is some point in the ith subinterval when we divide the interval between a and b into n equal subintervals

$x_1, x_2, \ldots, x_n$

e.g. Average value of $f(x)$ on $a \le x \le b$ (f is cont' on $a \le x \le b$)

$V = \lim_{n\to\infty}$ [Average of $f(x_i)$ at n evenly spaced sample points $x_1, \ldots, x_n$ on the interval]

$= \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} f(x_i)$

$$= \cancel{\frac{1}{n}\sum_{i=1}^{n} f(x_i)}$$

$$= \lim_{n\to\infty} \frac{1}{b-a}\,\Delta x \sum_{i=1}^{n} f(x_i) \qquad \Delta x = \frac{b-a}{n}$$

$$= \frac{1}{b-a} \lim_{n\to\infty} \Delta x \sum_{i=1}^{n} f(x_i)$$

$$= \frac{1}{b-a} \int_a^b f(x)\,dx$$

<u>Remark</u>: Now the ~~aver~~ integral of the differences from the average is 0:

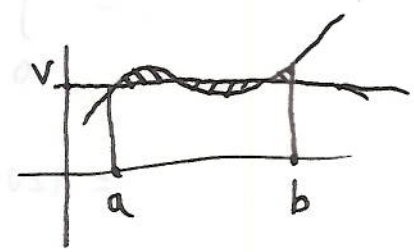
$$\int_a^b (f(x) - V)\,dx$$

$$= \int_a^b f(x)\,dx - \int_a^b V\,dx$$

$$= \int_a^b f(x)\,dx - V(b-a)$$

$$= \int_a^b f(x)\,dx - \int_a^b f(x)\,dx$$

$$= 0$$



Area (▧) − Area (▨) = 0

"Future value" and "Present value"

Example: Money is transferred continuously into an account at a rate of \$1,200 /year and accumulates interest at the annual rate of ~~18%~~ 8% compounded continuously

What is in the account after 2 years?

(Recall this means that the rate of increase due to interest is (0.08)[current sum]/y

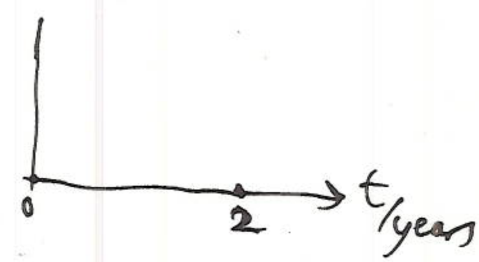
(so initial sum of P becomes

$$S(t) = Pe^{0.08t} \quad \text{after } t \text{ years}$$

$$(S'(t) = 0.08Pe^{0.08t}$$
$$= (0.08)S(t))$$

Approximate with Riemann sums:

Divide $0 \le t \le 2$ into n subintervals

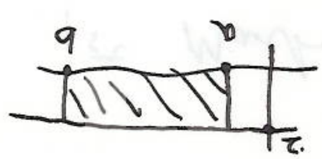


$\frac{2}{n}1200$ added to the account in ~~each~~ the i^{th} subinterval – pretend that this $\frac{2}{n}1200$ is all added at once, at some time t_i in the i^{th} subinterval.

$$\int_a^b 2\,dx = 2(b-a)$$

$$\int 2\,dx = 2x$$

$$\int_a^b 2\,dx = \left[\int 2\,dx\right]_a^b = \left[2x\right]_a^b = 2b - 2a = 2(b-a)$$



Then once interest is considered, this $\frac{2}{n}1200$ will contribute $\left(\frac{2}{n}1200\right)e^{0.08(2-t_i)}$ to the sum after 2 years

So we estimate the final sum as

$$S_n = \sum_{i=1}^{n}\left(\frac{2}{n}1200\right)e^{0.08(2-t_i)}$$

$$= \frac{2-0}{n}\sum_{i=1}^{n} 1200\, e^{0.08(2-t_i)}$$

so $S := \lim_{n\to\infty} S_n = \int_0^2 1200\, e^{0.08(2-t)}\,dt$

$$= \int_0^2 1200\, e^{(0.08)(2)} e^{-0.08t}\,dt$$

$$\frac{d}{dt}e^{\lambda t} = \lambda e^{\lambda t}$$

$$\int e^{\lambda t}\,dt = \frac{1}{\lambda}e^{\lambda t} + c$$

$$= 1200\, e^{0.16}\int_0^2 e^{-0.08t}\,dt$$

$$= 1200\, e^{0.16}\left[\frac{1}{-0.08}e^{-0.08t}\right]_0^2$$

$$= \frac{-1200}{0.08}e^{0.16}\left(e^{-0.16}-1\right)$$

$$\approx 2600$$

<u>Generally</u>: If money is transferred continuously to an account at a rate $f(t)$ during $0 \le t \le T$, and if interest is accumulated at rate r, the "<u>future value</u>" FV is the amount in the account at $t = T$

same argument:

$$FV = \lim_{n\to\infty} \Delta t \sum_i f(t_i)e^{r(T-t_i)}$$

$$= \int_0^T f(t)e^{r(T-t)}\,dt = e^{rT}\int_0^T f(t)e^{-rt}\,dt$$

The Present value, PV, is the amount which, were it deposited now and accumulated interest at rate r, it would worth, at $t=T$, FV

i.e. $PVe^{rT} = FV$

i.e. $PV = e^{-rT} FV$

$$= e^{-rT} e^{rT} \int_0^T f(t) e^{-rt}\, dt$$

$$= \int_0^T f(t) e^{-rt}\, dt$$