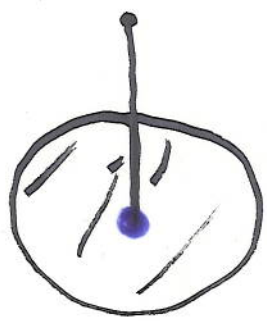
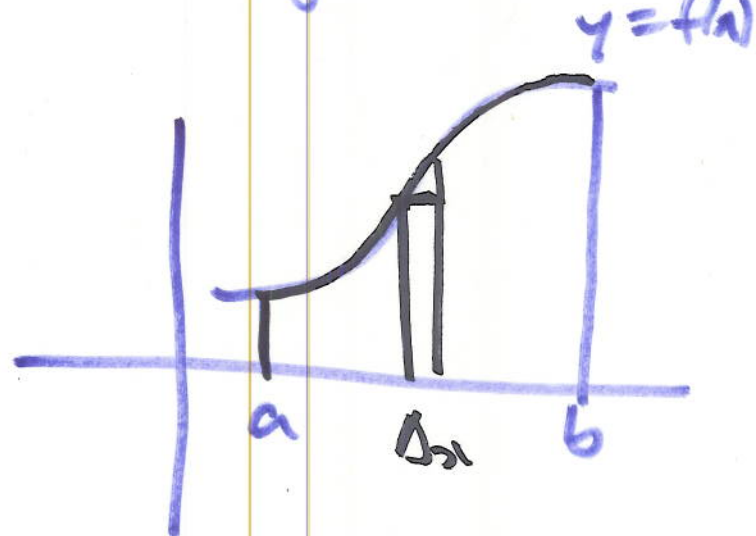
Lecture 29 More applications of integration.





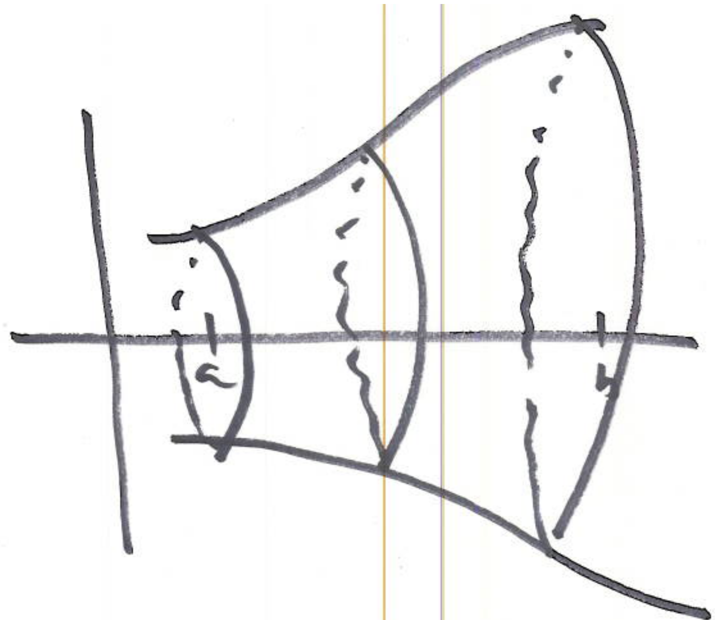
~~a~~ X section $\perp^r$ to x-axis is a circle of radius $f(x)$.

volume of a small piece of the solid is area of X-section X Δx

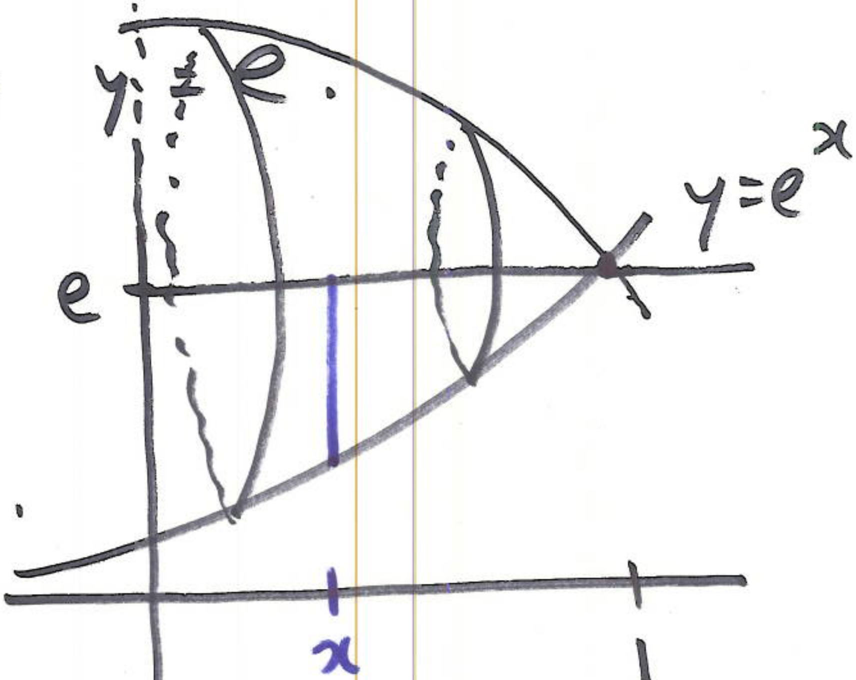
$$\pi (f(x))^2 \Delta x$$

$$\text{vol solid} \simeq \sum \pi (f(x))^2 \Delta x$$

$$\text{vol} = \int_a^b \pi (f(x))^2 dx.$$



Ex. Find the volume of the solid formed by rotating the graph of $y=e^x$ from $x=0$ to $x=1$ around the line $y=e$.



X-section is a circle of radius $e-e^x$

$$\text{volume} = \int_{x=0}^{1} \pi\left(e-e^x\right)^2\,dx$$

$$\text{vol} = \pi \int_0^1 \left(e^2 - 2ee^x + (e^x)^2\right) dx$$

$$(e^x)^2 = e^{2x}$$

$$= \pi \left[e^2 x - 2ee^x + \frac{1}{2} e^{2x} \right]_0^1$$

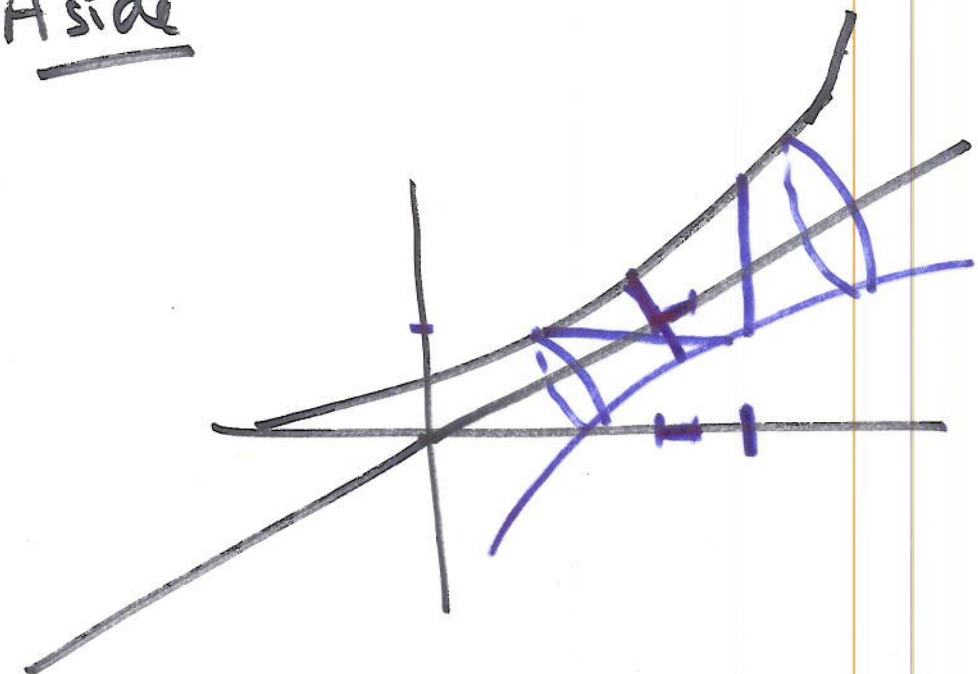
$$= \pi \Big(e^2 - 2ee^1 + \frac{1}{2} e^2$$

$$- \left(e^2 \cdot 0 - 2ee^0 + \frac{1}{2} e^0 \right) \Big)$$

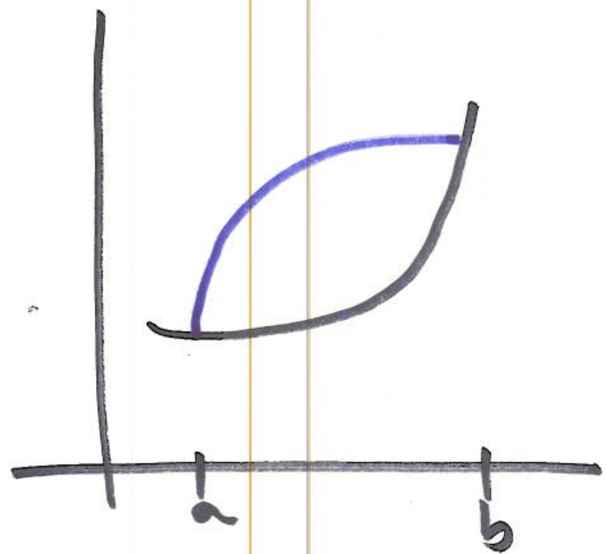
$$= \pi \left(-\frac{1}{2} e^2 + 2e - \frac{1}{2} \right).$$

Another application: average value of a function.

Aside

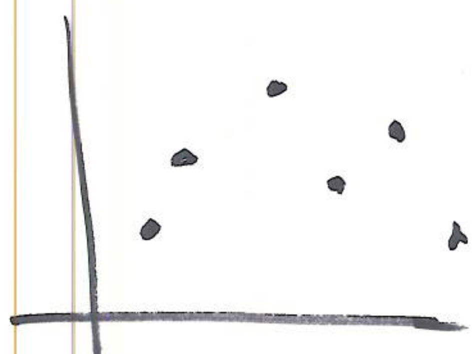


rotating around an axis
not parallel to one of the
coordinate axes - hard!

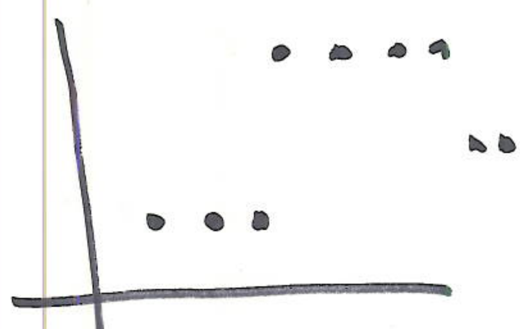


average value of $y_1, \dots, y_n$ is

$$\frac{y_1+y_2+\dots y_n}{n}$$

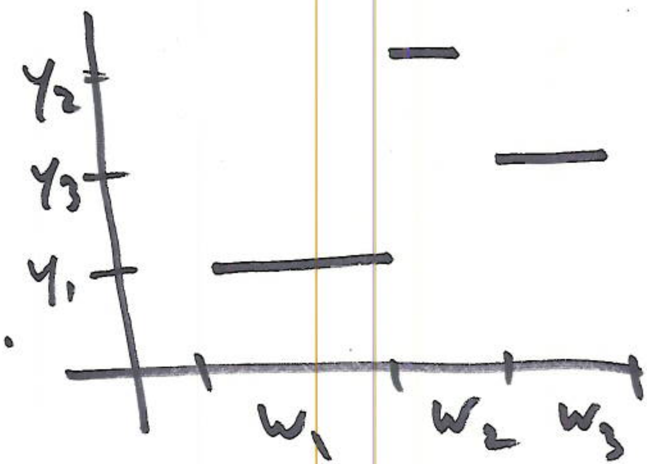


If the values of the data y_i has weight w_i then avg

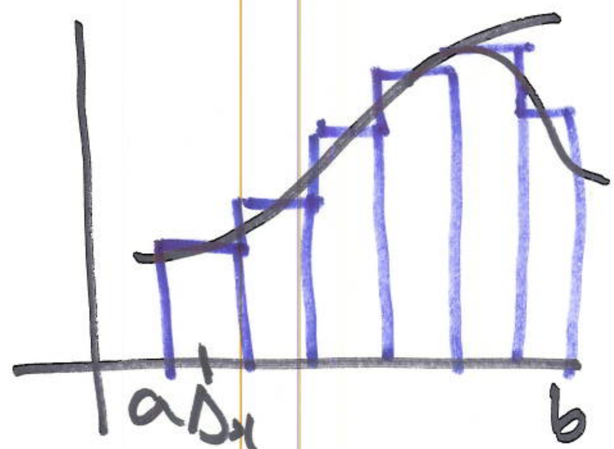


$$\frac{y_1w_1+y_2w_2+\dots+y_nw_n}{w_1+w_2+\dots+w_n}$$

The weights might be the length of an interval.



values of the function $f(x)$, assume constant on Δx



then the avg $= \dfrac{\sum f(x^*)\,\Delta x}{\sum \Delta x}$

$\sum \Delta x = b-a$

$$\text{average} = \lim_{\Delta x \to 0} \frac{\sum f(x^*)\,\Delta x}{\sum \Delta x}$$

$$= \frac{1}{b-a} \lim_{\Delta x \to 0} \sum f(x^*)\,\Delta x$$

$$\boxed{\begin{array}{l}\text{avg value} \\ \text{of } f(x) \\ \text{on } [a,b].\end{array} = \left\{ \frac{1}{b-a}\int_a^b f(x)\,dx \right\}}$$

$$\sum_{i=1}^{n} \frac{b-a}{n} = \frac{b-a}{n} + \frac{b-a}{n} + \dots \frac{b-a}{n}$$

$$= n\left(\frac{b-a}{n}\right) = b-a.$$

<u>Ex</u>. Find the avg. value of $f(x) = \sin(x)$ on $[0,\pi]$.

$$\text{avg} = \frac{1}{\pi - 0}\int_0^\pi \sin(x)\,dx$$

$$= \frac{1}{\pi}\Big[-\cos(x)\Big]_0^\pi$$

$$= \frac{1}{\pi}\left(-\cos(\pi) - (-\cos(0))\right)$$

$$\text{avg} = \frac{1}{\pi}(1+1)$$

$$= \frac{2}{\pi}.$$