Linear discrete dynamical systems
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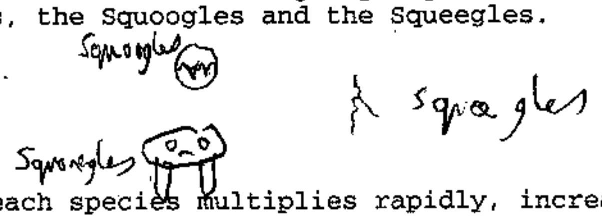
Example - population dynamics:
The caves on mars contain intriguing vegetation and three species of fauna: the Squongles, the Squoogles and the Squeegles.



Left alone, each species multiplies rapidly, increasing 30% per Martian year.

However, the Squeegles sometimes consume the Squoogles, who hunt the Squongles, who are peaceful but tend to accidentally step on the Squeegles.

A scientific mission is planned to investigate one of the caves. Its current population is estimated to be 0.8m Squongles, 0.6m Squoogles, 0.4m Squeegles.

In this cave, on average a Squeegle kills off a Squoogle once every two years. So the Squoogle population is reduced each year by 50% of the Squeegle population. Similarly for Squoogles killing Squongles and Squongles killing Squeegles.

Let $\underline{p}_t$ be the 3-vector of populations in year t

$\underline{p}_0 = \begin{pmatrix} 0.8 \\ 0.6 \\ 0.4 \end{pmatrix}$ population of squongles after a year $(t=1)$ is 1.3 times current population of squongles − 0.5 times current population of squoogles

You are evil, and work for the bioweapons division of your government, which does not want the existence of life on mars revealed to the public. Killing off large numbers of Martians is difficult, but you could introduce further Squongles, Squoogles and Squeegles from other caves.

Using your eigen-fu, can you find a way to reduce the population sufficiently that astro-agents with pointy sticks will suffice to finish off the rest?

The Squongles have started deliberately trying to stamp on Squeegles, and now each kill 1 Squeegle per year. How does this change things?

so $\quad \underline{p_1} = \begin{pmatrix} 1.3 & -0.5 & 0 \\ 0 & 1.3 & -0.5 \\ -0.5 & 0 & 1.3 \end{pmatrix} \underline{p_0}$

$$\underline{p_2} = \underbrace{\begin{pmatrix} 1.3 & -0.5 & 0 \\ 0 & 1.3 & -0.5 \\ -0.5 & 0 & 1.3 \end{pmatrix}}_{= A} \underline{p_1} = AA\underline{p_0} = A^2 \underline{p_0}$$

$$\underline{p_t} = A^t \underline{p_0}$$

$$\chi_A(\lambda) = \det(\lambda I - A) = \det \begin{pmatrix} \lambda - 1.3 & 0.5 & 0 \\ 0 & \lambda - 1.3 & 0.5 \\ 0.5 & 0 & \lambda - 1.3 \end{pmatrix}$$

$$= (\lambda - 1.3)^3 + (0.5)^3 = 0$$

$$(\lambda - 1.3)^3 = -(0.5)^3$$

so $\lambda - 1.3 = -0.5$ will be a solution

i.e $\lambda = 0.8$ only (real) solution

$$A \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.8 \\ 0.8 \\ 0.8 \end{pmatrix} = 0.8 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

so ~~this~~ is $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is a 0.8 e-vector.

and hence so is $\begin{pmatrix} t \\ t \\ t \end{pmatrix}$ for any $t \neq 0$

so if $\underline{\rho_0} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

then $\underline{\rho_t} = A^t \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$$= 0{\cdot}8^t \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$0{\cdot}8^t \xrightarrow[t \to \infty]{} 0$$