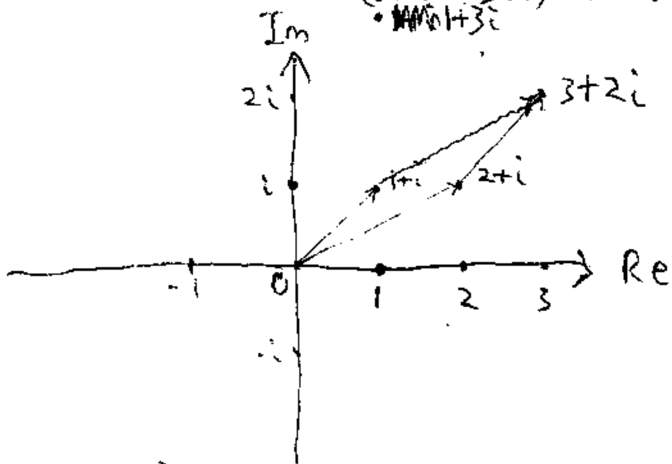
$a + bi \qquad i^2 = -1$

$$(a+bi)(c+di) = ac + adi + bci + bdi^2$$
$$= (ac - bd) + (ad + bc)i$$



addition of complex numbers $\longleftrightarrow$ vecter addition

"Argand diagram"
"Complex plane"

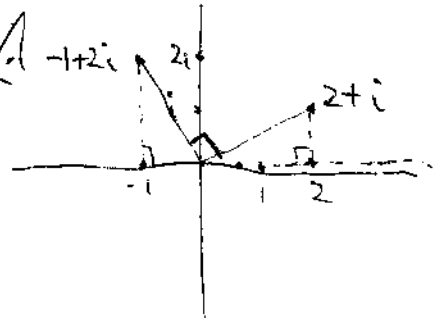
$$(1+i)(2+i) = 2 + i + 2i + i^2$$
$$= 1 + 3i$$

$i^2 = -1 \qquad i^3 = i^2 i = -i \qquad i^4 = i^2 i^2 = (-1)^2 = 1$

$(-i)^2 = (-1)^2 i^2 = i^2 = -1$

(generally $(-z)^2 = ((-1)z)^2 = (-1)^2 z^2 = z^2$

~~Remark~~



$i(2+i) = -1 + 2i$

Remark: For any complex $z = a + bi$,

$$zi = iz = -b + ai$$

so iz is what you get if you rotate z by $\pi/2$ counterclockwise around 0

Polar form

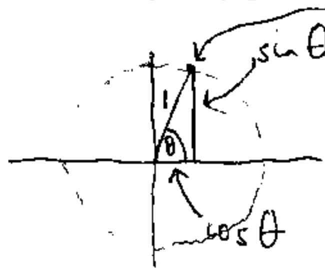
Definition: For θ a real number,

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$\cos\theta = Re(e^{i\theta})$
$\sin\theta = Im(e^{i\theta})$

$=$ [the point on the unit circle at angle θ counterclockwise to the real axis]

(also $\cos\theta =$ [illegible] think about it)



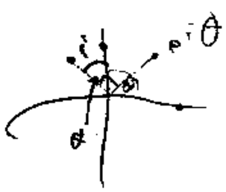
$$\sqrt{(\cos\theta)^2 + (\sin\theta)^2} = \sqrt{1} = 1$$

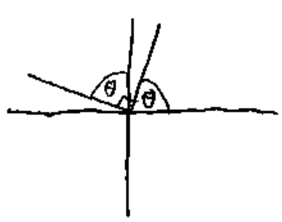
Lemma: For any real θ and ϕ

$$e^{i\theta} \cdot e^{i\phi} = e^{i(\theta+\phi)}$$

Proof: If $\phi = 0$, so $e^{i\phi} = e^0 = 1$, then $e^{i\theta}e^{i\phi} = e^{i\theta} = e^{i(\theta+0)}$

If $\phi = \pi/2$, so $e^{i\phi} = i$, then $e^{i\theta}e^{i\phi} = e^{i\theta}i =$ [$e^{i\theta}$ rotated by $\pi/2$] $= e^{i(\theta+\pi/2)}$





So multiplying $e^{i\theta}$ with 1 and with i acts as rotation by θ

so if $z = a + bi$

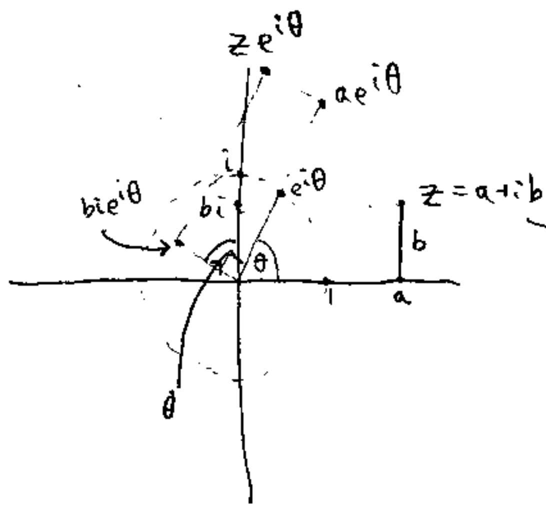
then $e^{i\theta} z = e^{i\theta}(a+bi) = e^{i\theta}a + e^{i\theta}bi$

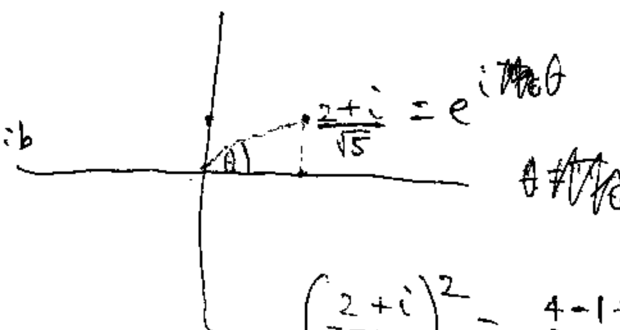
$= [a \text{ rotated by } \theta] + [bi \text{ rotated by } \theta]$

$= [a+bi \text{ rotated by } \theta]$

$= [z \text{ rotated by } \theta]$

so $e^{i\theta}e^{i\phi} = [e^{i\phi}$ rotated by $\theta] = e^{i(\theta+\phi)}$





$$\left(\frac{2+i}{\sqrt{5}}\right)^2 = \frac{4-1+4i}{5} = \frac{3+4i}{5} = e^{i2\theta}$$