$\boxed{11/3/3}$ n-space

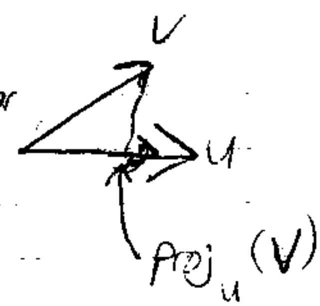
Def'n $\mathbb{R}^n$ &

If u is a unit vector and v is a vector

$\text{proj}_u(v) = (v \cdot u)u$

and if u is any vector

let $\hat{u} = \frac{u}{\|u\|}$ unit vector



define $\text{proj}_u(v) = \text{Proj}_{\hat{u}}(v)$

$= \left(v \cdot \frac{u}{\|u\|}\right)\frac{u}{\|u\|} = \frac{v \cdot u}{\|u\|^2} u$

Plane

unit vector → $n \cdot x = c$ ← min distance of origin to the plane

↑ linear equation

Parametric form of a plane

If x_0, x_1 and x_2 points on a plane P

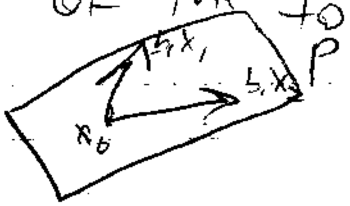
let $b_1 = x_1 - x_0$

$b_2 = x_2 - x_0$

and if b_1 and b_2 are not collinear

then every point of the plane is of the form $\boxed{x_0 + sb_1 + tb_2}$

← s, t are real numbers



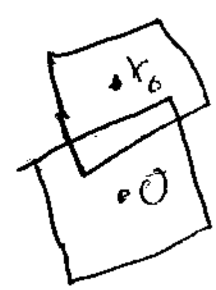
Geometry of linear Systems

Let A be $m \times n$ real matrix
let $a_1, \ldots, a_m$ be the rows
Then $Ax = 0 \iff a_1 \cdot x = 0$ and $a_2 \cdot x = 0$
and ... and $a_m \cdot x = 0$
$\iff x$ is orthogonal (normal)
to $a_1, \ldots a_m$)

Solving $Ax = 0$ (e.g. by Gaussian elimination)
gives a parametric form for this set
of Solutions

$$t_1 u_1 + \ldots + t_s u_s$$

The set of solutions to $Ax = b$
is the translate of the set of solutions
to $Ax = 0$ by any solution x_0 to $Ax = b$
ie. $A(x - x_0) = 0$ iff $Ax = b$
if $Ax_0 = b$



Cross Product $\mathbb{R}^3$

v, w vectors in $\mathbb{R}^3$ (AKA 3 space)

$$v = (v_1, v_2, v_3) \quad w = (w_1, w_2, w_3)$$

$$v \times w = \left(\begin{vmatrix} v_2 & v_3 \\ w_2 & w_3 \end{vmatrix}, - \begin{vmatrix} v_1 & v_3 \\ w_1 & w_3 \end{vmatrix}, \begin{vmatrix} v_1 & v_2 \\ w_1 & w_2 \end{vmatrix} \right)$$

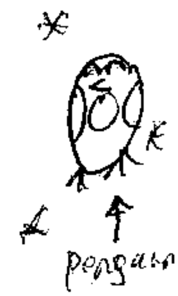
The fundamental property

For any u in $\mathbb{R}^3$

$$u\cdot(v\times w) = \det\begin{pmatrix} | & | & | \\ u & v & w \\ | & | & | \end{pmatrix}$$

Proof: cofactor expansion down first column

$$\det\begin{pmatrix} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{pmatrix} = u_1\begin{vmatrix} v_2 & w_2 \\ v_3 & w_3 \end{vmatrix}$$



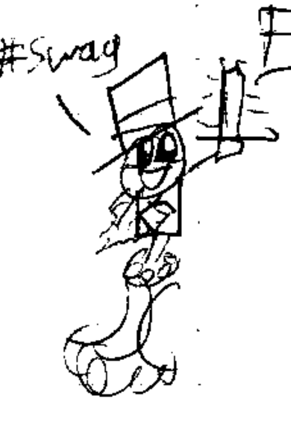
$$u(v\times w) = \begin{matrix} -u_2 \\ +u_3 \end{matrix}\Big| \quad \Big|$$

So if v and w are collinear then $v\times w = 0$
and for any v, w

$$v\cdot(v\times w) = 0 = w\cdot(v\times w)$$

So $v\times w$ is normal to v and to w

Now suppose v, w not collinear



Fact: For any u, v, w in $\mathbb{R}^3$

$$\det\begin{pmatrix} | & | & | \\ u & v & w \\ | & | & | \end{pmatrix} = \text{SignedVol}(\text{the parallelepiped of } u, v, w)$$

AKA a 3-D rhombus

$$u\cdot(v\times w) = \det\begin{pmatrix} | & | & | \\ u & v & w \\ | & | & | \end{pmatrix}$$

$\hat{i}$ is a column vector $\in \mathbb{R}^3$

$$\begin{pmatrix} | & | & | \\ \hat{i} & \hat{j} & \hat{k} \\ | & | & | \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$